VERTICAL MOVEMENTS FROM LEVELLING, GRAVITY AND GPS MEASUREMENTS

N. Hatjidakis, D. Rossikopoulos
Department of Geodesy and Surveying, Faculty of Engineering
Aristotle University of Thessaloniki, email: nhatji@topo.auth.gr, rossi@topo.auth.gr

Abstract

Height differences from geometric leveling, gravity and GPS measurements carried out from 1979 until now in the network of Volvi area are systematically analyzed. Various methods and algorithmic techniques based on integrated approach are rigorously applied in order to extract a more reliable estimation of the vertical components of the deformation field. Signals, i.e. the gravity field parameters, their variations in time and the vertical displacements are treated in a combined analytical-stochastic approach. Finally the interpretation of the results at the interesting area of the Volvi Lake is attempted.

1. Introduction

The estimation of displacement and especially the height differences at Volvi area is very important in Greece due to its seismic activity. Volvi area is located at northern Greece (Central Macedonia) and 40Km from the city of Thessaloniki. Volvi take up an area of 40X45km including two lakes (Lagada and Volvi) and is considered as a geodynamically interesting area. The largest earthquake took place in May 23, 1978 (Ms= 5.8), June 19, 1978 (Ms= 5.2) and June, 20 1978 (Ms= 6.2) followed by a series of postseismic activity affected socially and financially the city of Thessaloniki.

In order to study the geodynamical behaviour and investigate the crustal movements a geodetic and gravimetric network was established with 16 and 26 pillars respectively. Classical measurements are available for the epoch 1979, gravity measurements took place in 1979 and 1999 while GPS measurements performed in 1994, 1995, 1996 and 2003.

The object of this presentation is the simultaneously analysis of GPS baselines with classical geodetic observations (height differences from levelling) and physical data related to the gravity field (absolute gravity values) with models of integrated geodesy (e.g.[4]), having as main purpose the estimation of orthometric heights in different epochs.

2. The integrated approach

Integrated geodesy in time deals with the analysis of the above observations for the study of network geometry and its variation with time, when these observations depend on the gravity field of the earth and its temporal variation.

Integrated adjustment has been introduced as a rigorous adjustment of observations with both geometric and gravimetric information using precise mathematical goals. Furthermore, integrated geodesy is a method for the adjustment of observations depending not only on discrete parameters but also on unknown functions.

Parameters appearing in the mathematical model can be treated as: a) deterministic b) stochastic c) analytical – stochastic way (e.g. [9], [3]) and these leads to an adjustment with signals. Applications of relevant concepts in the estimation of orthometric heights from GPS baselines have been presented by [6], [7].

Geodetic-gravimetric network deals with GPS baselines, leveling and gravity observations in different epochs so orthometric heights are much easier to calculate. The difficult part of analyzing simultaneously physical and geometrical observations in time is to find out the gravity field behavior. This specifies the usefulness of time-space covariance model which help us to understand the previous thought.

The local gravity behaviour consists a covariance gravity model (e.g. [8]). The most interesting field is the choice of local covariance model which includes contrasts between correlation lengths. As we can see from the matrix below

	Variance σ_g^2 (mgal^2)	Correlation length d(m)
Exponential	777.38	21066.794
Reilly	719.56	36660.197
Moritz	789.10	48074.483
Poisson	763.40	69183.422

Table 1. Parameters of the covariance model

the most proper choice is the exponential model since correlation length is more suitable for a small area.

2.1 Gravity and geometry deformation

The behaviour of the gravity field is the interesting part of this study. Calculating the changes between the corrections in the geoid undulations from 1994 until 2003 and gravity anomaly perturbations between 1979 and 1999 epochs using a time-space covariance (exponential) model proved that no signal changes are calculated from 1979 until 2003. The adjustment with the integrated model can be presented using the following general formula

$$\begin{bmatrix} \mathbf{b}_0 \\ \mathbf{b}_1 \\ \vdots \\ \mathbf{b}_n \end{bmatrix} = \begin{bmatrix} \mathbf{A}_0 & 0 & \dots & 0 \\ 0 & \mathbf{A}_1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbf{A}_n \end{bmatrix} \begin{bmatrix} \mathbf{x}_0 \\ \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_n \end{bmatrix} + \begin{bmatrix} \mathbf{G}_0 & 0 & \dots & 0 \\ 0 & \mathbf{G}_1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbf{G}_n \end{bmatrix} \begin{bmatrix} \mathbf{s}_0 \\ \mathbf{s}_1 \\ \vdots \\ \mathbf{s}_n \end{bmatrix} + \begin{bmatrix} \mathbf{v}_0 \\ \mathbf{v}_1 \\ \vdots \\ \mathbf{v}_n \end{bmatrix} \tag{1}$$

The adjustment of the observations is carried out by applying the least squares principles

$$\sum_{\alpha=1}^{n} (\mathbf{v}_\alpha^T \mathbf{P}_\alpha \mathbf{v}_\alpha + \mathbf{s}_\alpha^T \mathbf{K}_\alpha^{-1} \mathbf{s}_\alpha) = \min \tag{2}$$

where $\mathbf{x}_i$ contains the deterministic parameters (orthometric heights), $\mathbf{s}_i$ contains all the stochastic parameters (gravity disturbances, geoid undulations) and $\mathbf{v}_i$ are the observation errors for each epoch. Furthermore, the indicator $(_\alpha)$ refers to t_α epoch, $\mathbf{P}_\alpha = \mathbf{Q}_\alpha^{-1}$ is the weight observation matrix at each epoch and $\mathbf{K}_\alpha^{-1}$ is the covariance matrix developed from signals at each epoch. The results of the above algorith is presented to the following table

	δg_{1979}	δg_{1999}	$\delta_t \delta g$ (mgal)	$\delta_t h$(cm)		δg_{1979}	δg_{1999}	$\delta_t \delta g$ (mgal)	$\delta_t h$(cm)
1	48.186	48.460	0.1	0.09	14	54.159	52.996	-0.4	-0.38
2	59.146	58.247	-0.3	-0.29	15	48.236	48.988	0.2	0.24

3	59.895	59.116	-0.3	-0.25	16	50.675	51.569	0.3	0.29
4	55.222	53.070	-0.7	-0.70	17	54.197	54.982	0.3	0.25
5	51.417	49.603	-0.6	-0.59	18	44.717	44.652	0.0	-0.02
6	45.886	46.164	0.1	0.09	19	49.026	50.227	0.4	0.39
7	55.308	54.953	-0.1	-0.12	20	53.257	52.858	-0.1	-0.13
8	56.138	54.801	-0.4	-0.43	21	48.425	48.378	0.0	-0.02
9	52.740	51.425	-0.4	-0.43	22	44.103	46.032	0.6	0.63
10	48.286	47.075	-0.4	-0.39	23	52.099	53.703	0.5	0.52
11	44.701	43.740	-0.3	-0.31	24	47.484	49.003	0.5	0.49
12	49.657	50.219	0.2	0.18	25	41.763	43.551	0.6	0.58
13	56.438	56.895	0.1	0.15	26	44.328	44.767	0.1	0.14

Table 2. Gravity changes perturbations between 1999 –1979 time period

The analytical formula between height and gravity disturbances is (e.g.[2])

$$\delta_t \delta g[\mathrm{mgal}] = 3.086 \delta_t h[\mathrm{cm}] \tag{3}$$

	1994	1995	1996	2003
40	-0.26			-0.17
33	-0.44	-0.15	-0.26	-0.13
56	-0.63		0.27	-0.18
51	-0.15	-0.03		-0.02
64	-0.27	-0.12	0.12	-0.13
44	-0.10	-0.01	-0.01	-0.02
54	-0.14	-0.01	-0.08	-0.05
77	-0.31	-0.08	0.14	-0.03
320	-0.33			-0.23
420	-0.59			0.09
46		-0.07	-0.03	
444		-0.03	-0.01	-0.05
45		-0.01	-0.02	
38		-0.08	-0.07	
37			-0.14	
50			-0.05	
530				0.02
520				0.04
540				0.08
310				0.07
620				0.03
max	-0.10	-0.01	0.27	0.09
mv	-0.3	-0.1	0.0	0.0

Table 3. Changes of geoid undulations δN between 1994 and 2003 time period

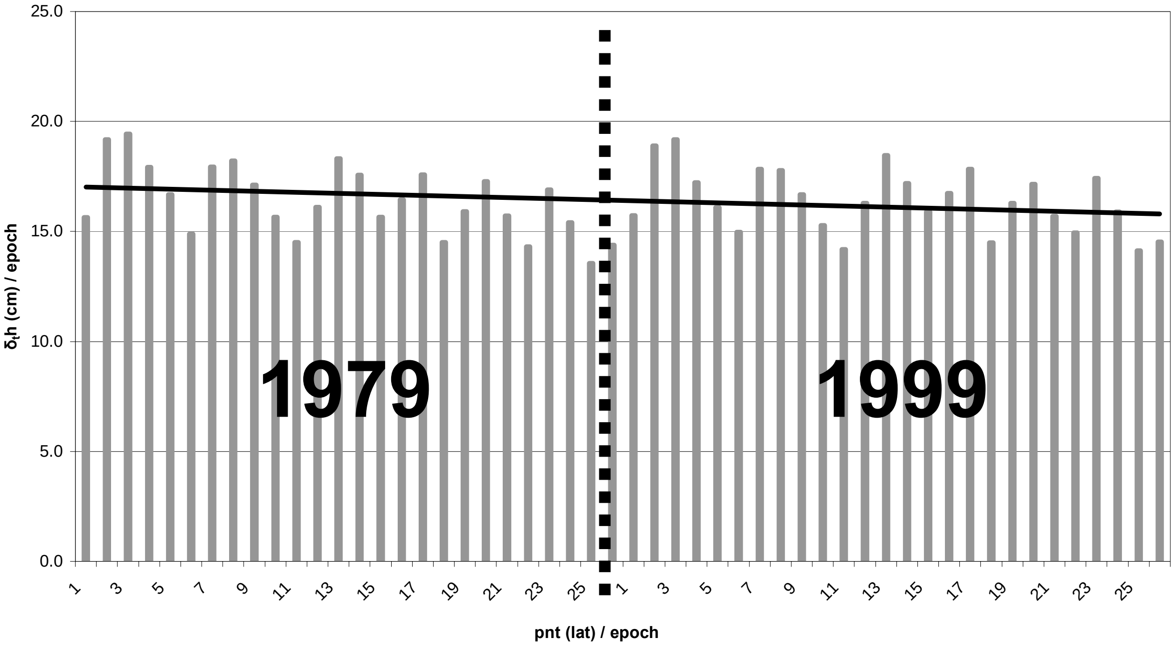

Figure 1. Absolute gravity values - without space-time covariance model

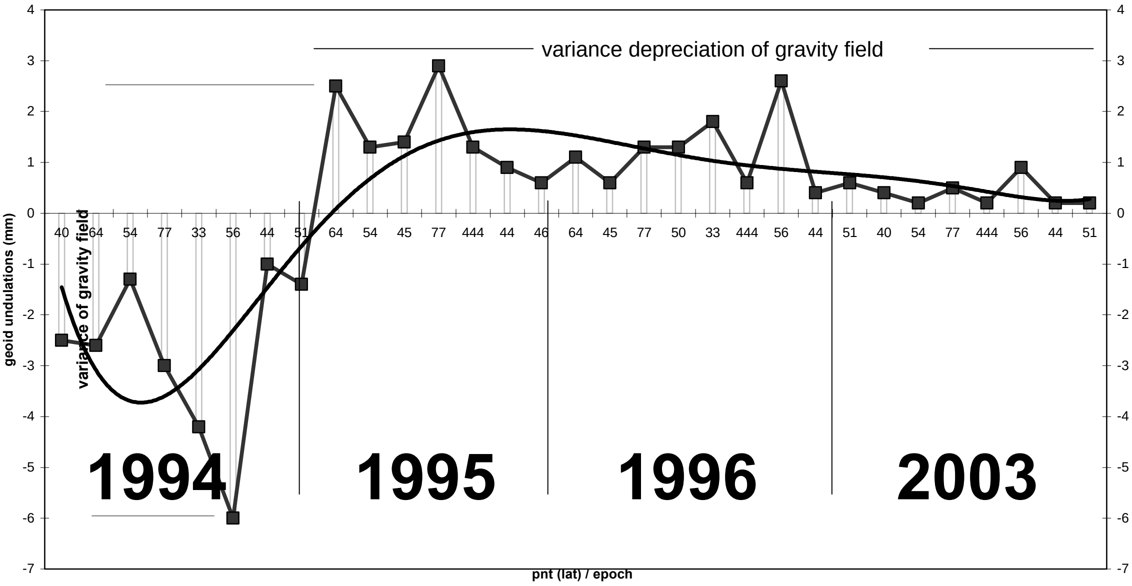

Figure 2. GPS baselines- signal connection with space-time covariance model
(9 yr (2003-1994))

In figure 2 variance depreciation of gravity field took place every nine (9) years. So the phenomeno of seismicity repeated every nine years, which is confirmed by historical seismological data [10]. Because of earthquakes took place in 1969,in 1978,in 1995 and in 2003 (every nine years) time – space covariance model become a useful prediction tool in Seismology.

y_b^t	1979	1994	1995	1996	1999	2003
Observations	LEV	GPS	GPS	GPS	GRAV	GPS
$\mathbf{v}^T\mathbf{P}\mathbf{v}+\mathbf{s}^T\mathbf{K}^{-1}\mathbf{s}=\min$	255.9546	26373.3589	36876.5758	38036.1584	33523.04	49130.5457
degrees of freedom f	2	129	57	36	26	207
variance $\hat{\sigma}^2$	127.9773	204.4446	646.9575	1057.9488	1298.3480	237.3456
standard deviation $\hat{\sigma}$	11.31	14.30	25.44	32.53	36.03	15.41

Table 4. Aposteriori adjustment parameters for 1979, 1994, 1995, 1996, 1999 and 2003 epochs

By calculating gravity disturbances or corrections of the geoid undulations the differences are too small, so the changes of absolute gravity observations are due to height differences. This simplifies the algorithmic problem and makes calculations be accessible.

2.2 Geometry deformation at fixed gravity

All observations (GPS baselines, gravity field data and levelling) can be analysed simultaneously even if they measured in different epochs. Signals **s** are unchangeably from 1979 until 2003 (Figure 1) but gravity measurements which took place in 1979 and 1999 are changed since points are moved. The adjustment with the integrated model can be presented

$$\begin{bmatrix}\mathbf{b}_0\\ \mathbf{b}_1\\ \vdots\\ \mathbf{b}_n\end{bmatrix}=\begin{bmatrix}\mathbf{A}_0 & 0 & \dots & 0\\ 0 & \mathbf{A}_1 & \dots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \dots & \mathbf{A}_n\end{bmatrix}\begin{bmatrix}\mathbf{x}_0\\ \mathbf{x}_1\\ \vdots\\ \mathbf{x}_n\end{bmatrix}+\begin{bmatrix}\mathbf{G}_0\\ \mathbf{G}_1\\ \vdots\\ \mathbf{G}_n\end{bmatrix}\left[\mathbf{s}\right]+\begin{bmatrix}\mathbf{v}_0\\ \mathbf{v}_1\\ \vdots\\ \mathbf{v}_n\end{bmatrix} \tag{4}$$

where $\mathbf{x}_i$ contains the deterministic parameters (orthometric heights), **s** contains all the stochastic parameters (gravity disturbances, geoid undulations common for all epochs) and $\mathbf{v}_i$ are the observation errors for each epoch. The adjustment of the observations is carried out by applying the least squares principles

$$\sum_{\alpha=1}^{n}\mathbf{v}_\alpha^T\mathbf{P}_\alpha\mathbf{v}_\alpha+\mathbf{s}^T\mathbf{K}_s^{-1}\mathbf{s}=\min \tag{5}$$

which leads to the best linear unbiased estimation for the deterministic parameters **x** and the best linear predictions for the stochastic ones **s**, **v**.
The results of the above algorith is presented to the following table

	1979	1994	1995	1996	1998	2003	Height differences (mm/yr)
40		200.469 2.27				200.478 2.05	1.00
64	210.653 0.02	210.567 1.56	210.522 4.97	210.611 7.99	210.675 4.05	210.645 1.25	0.33
54	81.203 0.02	81.218 1.44	81.154 3.99	81.158 4.97	81.153 4.94	81.134 0.89	4.00
37				212.137 5.31			
45			90.801 3.71	90.762 2.42	90.825 3.22		-39.00
77	254.238	254.299	254.283	254.323	254.409	254.377	5.79

	0.02	1.95	1.76	3.51	3.77	0.64	
46	90.419 0.02		90.313 2.15	90.296 7.97	90.322 4.77		-7.24
50				255.039 4.73			
33		381.950 3.02	382.007 5.54	381.914 5.60	381.923 3.75	381.962 1.67	1.33
444			83.454 3.16	83.435 7.34	83.401 4.02	83.400 2.91	-6.75
56		566.601 1.80		566.814 2.83	566.596 4.31	566.629 2.05	3.11
44		42.666 3.88	42.708 4.71	42.645 0.79	42.560 2.10	42.563 6.15	-11.44
51		90.370 4.66	90.427 2.02	90.424 4.65	90.380 4.00	90.349 3.27	-2.33
310						364.904 4.55	
320		272.216 3.17				272.18 6.01	-4.00
420		538.914 1.40				538.927 2.34	1.44
520						192.902 5.72	
530						66.891 6.17	
540						447.562 2.38	
620						130.036 0.87	
38			234.138 0.6	234.096 5.74	234.191 3.25		-42.00

Table 5. Orthometric Heights and standard deviation for each point per epoch

The mean height difference is 6.8mm/yr. Mean height differences calculated near to 5.7mm/yr from other scientific jobs developed in Volvi area.

y_b^t	1979	1994	1995	1996	2003
$\mathbf{v}^T\mathbf{P}\mathbf{v}+\mathbf{s}^T\mathbf{K}^{-1}\mathbf{s}=\min$	141.2117	26373.3589	36879.5758	38086.1584	49130.5457
f	2	139	67	49	223
$\hat{\sigma}^2$	282.4234	189.7364	550.3967	777.2685	220.3163
$\hat{\sigma}$	11.31	13.77	23.46	27.88	14.84

Table 6. Aposteriori adjustment parameters for 1979, 1994, 1995, 1996, 1999 and 2003 epochs

In figure 3 vertical movements are presented.

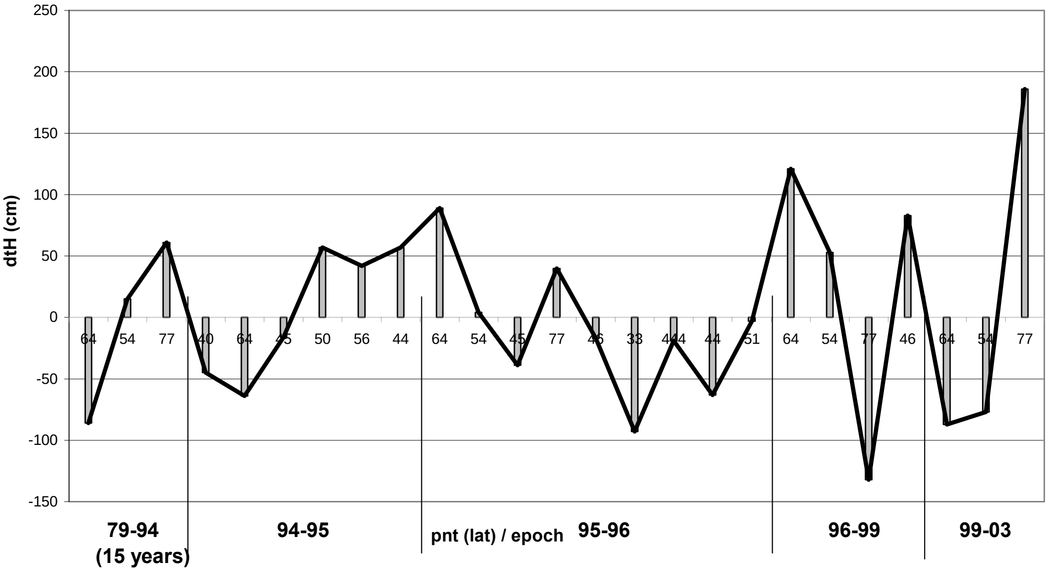

Figure 3. Height differences in cm (1979 - 2003) / pnt (lat) / epoch

4. Conclusions

We will define integrated geodesy as a powerful mathematical tool which manipulates physical and geometrical observations measured in different epochs making signal connection in space-time by the covariance gravity model. In the present study space-time covariance model help us to understand gravity field behaviour in Volvi area. Height differences calculated with high accuracy. Also, the biggest height differences occurred near to the points 54, 64, 46, 77 which indicates the epicentre of the earthquakes. We would like to point out that in the near future a covariance model will be attempt to replace the classical exponential model in volvi area which has the formula

$$K(S,z) = \sigma_g^2 e^{-\frac{2kM}{(R+z)^3}\sin^2\varphi S} \tag{6}$$

where $b = \frac{2kM}{(R+z)^3}\sin^2\varphi = 0.3083$ [1],[5], [11]is a mean constant value in the whole area.

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