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Calculating a geoid model for Greece using gravity and GPS observations

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Abstract

The main goal of the study is the calculation of a high resolution dataset that models the geoid for Greece using several kind of data collected by the Hellenic Military Geographical Service (HMGS). In situ gravity measurements and GPS/levelling on triangulation points plays a central role in the formation of covariance and cross-covariance functions used for the calculation of high frequency residuals. Also global models such as EGM2008 and EIGEN-6C4 contribute in the analysis of collected data and in the removal of low frequencies. A database of older gravity measurements completes and guarantees the data coverage of the whole region leading to a high resolution exported product. The whole project is based on the Remove – Compute – Restore (RCR) technique and the Least Squares Collocation (LSC) method is used at its core during the computation of the residual geoid height. In order to fulfill the RCR technique topographic corrections have been calculated on each measured point and the indirect effect has been computed for the total region. Rasters of the above have been extracted for visualization and analysis. The final product has been transformed through a parametric model for orthometric height adaption. Several scripts have been developed in Matlab and Python for the reckonings as no commercial or scientific software was used. Data combination and visualization in raster format has been made using the ESRI ArcGIS software. The study concludes to three different beta geoid height models depending on the RCR usage for further discussion and which will be evaluated in the light of new data collection.

I. INTRODUCTION

A. *Scientific Background*

The Geoid is essentially the real shape of the Earth, without topographic and atmospheric masses. The Geoid is defined as the equipotential surface of the Earth’s gravity field which coincides with the sea surface in the absence of disturbing factors like tsunamis, ocean currents, salinities, wind, etc., and it extends through the continents (Vaníček and Krakiwsky 1986). Though the geoid is much smoother than the actual earth surface, unlike the ellipsoid, it is still too complicated to serve as the computational surface on which to solve geometrical problems, but it is suitable as a vertical datum. Determination of a geoid model requires extensive gravitational measurements and computations.

A geoid model is required to define a national height or vertical datum. Precise geoid models have experienced an unprecedented demand due to the rapid development of GPS/GNSS technologies. Conversion of ellipsoidal height to orthometric height, which is more useful, requires an accurate geoid model. In spite of the sparse terrestrial gravity data of variable density, distribution and quality, this study set out to test the methodology to develop as accurately as possibly achievable, a high quality geoid model of Greece.

B. *Area of interest*

Greece is located in the Southeast part of Europe and is consist of over 2000 islands and the southeast part of valcanian peninsula. In this study geoid model refers mostly to continental part of Greece from Thrace to Northern Peloponnese. ($37.2^o<\lambda<41.2^o$, $19.3^o<\phi<26.7^o$)

II. DATA COMPILATION

A. *Measurements*

There is a gravity database with 8998 absolute gravity values in whole Greek territory. These values had not accurate coordinates and they were not surveyed on the ground. These measurements were took place at least 25 years ago. Although they present accurate values when they are tested with recent measurements.

There were 693 absolute gravity values at triangulation points in the central Greece. Measurements collected with gravimeter Lacoste & Romberg from 2000 to 2007 with theoretical accuracy better than 0.5 mgal.

There were 349 absolute gravity measurements at triangulation points using mostly relative gravimeter SCINTREX CG5. Measurements took place from 2013 to 2018 at selected locations at Northern and central Greece. Their theoretical accuracy is better than 0.2 mgal.

Finally, there were 1000 gps measurements at triangulation points with known orthometric height that took place in 2007. These points will be used as control points.

B. *Coordinate System*

The selected coordinate system for this research is WGS84 (G1674 edition). In order to be compatible with this system all gravity and GPS points need a propriate transformation from the epoch of the measurement to the selected coordinate system. Velocity model of Greek territory after Bitharis et al 2016 can improve the accuracy of transformation.

Absolute Gravity values refer to Potsdam system and converted to WGS84 gravity datum (subtracted 15 mgal to every measurement), in order to be compatible with the coordinate system.

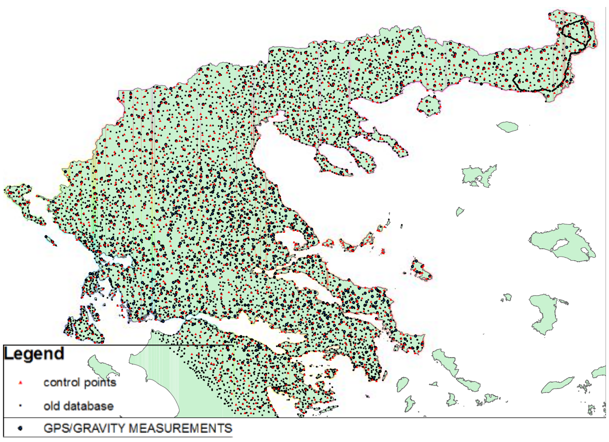

Figure 1. Data used for computation and control points over the area of interest. Bigger black dots refer to simultaneous GPS and gravity measurements to triangulation points from 2000 to 2018,smaller black dots refer to old gravity measurements from 1970 to 1995 and red dots refer to GPS measurements to triangulation points at 2007 that used as control points.

C. *Digital Terrain Model and Seafloor Topography*

The used 5m DTM derived from Hellenic Military Geographical Service (HMGS) originated from combined photogrammetric methods and height measurements. From this DTM three smoother DTMs were produced. (30m, 50m and 100m resolution).

D. *Global Geopotential Model (ggm)*

Three ggm were tested in Greek territory for this research: EGM2008 (Pavlis et al, 2008), EIGEN6C4 (Forste et al, 2015) and GECO (Gilardoni et al, 2015), until order and class 2190. These models were tested to 1089 points both to geoid undulations and free air gravity anomalies and in 8998 points only at free air gravity anomalies. The whole test was made in MATLAB code (GRAVSynth) (Papanikolaou & Papadopoulos, 2015) where spherical harmonic synthesis was built for each ggm at all points and compared calculated values to measured ones. In order to avoid outliers at measured values maximum differences were established as 2m at geoid undulation and 60 mgal at free air gravity anomaly. Finally, the routine calculated statistics and the systematic difference (bias) between measured and calculated geoid undulation after equation:

$$Nlev = Nggm + bias + u \qquad (1)$$

Where Nlev measured geoid undulation (N=h-H)
Nggm calculated geoid undulation
bias systematic difference
u random error

Bias showed the difference of the beginning of height system used at ggm compared to national height system.

Table 1. Comparison of GGM

GGM	1049 triangulation points		
	RMS (N) meter	Bias(N) meter	RMS (Δg) mgal
EGM2008	0.2305	-0.4023	25.3934
EIGEN6C4	0.2252	-0.4185	25.1846
GECO	0.2311	-0.4194	25.0164

From Table 1 it is obvious that all three models are in good agreement with measured data. EIGEN6C4 was a little more accurate from others and is the selected GGM.

III. METHOLOGY

A. *Reduction Scheme*

A gravity anomaly, which is the difference between gravity on the geoid and that on the reference spheroid, is produced by mass distributions that cause the geoid to deviate from the spheroid. Land measurements are made above sea level; measured gravity must then be reduced to the sea-level equivalent before an anomaly can be obtained by subtracting a value for normal gravity on the spheroid. The reduction scheme for Absolute gravity values in this research includes:

1) *Geographic latitude corrections:* All gravity data were reduced to the ellipsoid of GRS 1980 while WGS84 adopted as the gravity datum so that is retained consistency with the HMGS gravity measurements, using the closed Somigliana's formula (Somigliana 1930):

$$Gmod = ge * \frac{1+ksin^2\varphi}{\sqrt{1-e^2sin^2\varphi}} \quad (2)$$

where ge=978032.67714 mgal
k=0.00193185138639
e^2=0.00669437999013
ϕ latitude of the station in decimal degrees.

2) *Free air reduction*: The simply free air gradient was used:

$$\delta gFA = -0.3086H \quad (3)$$

where H is the orthometric Height above sea level in meters

3) *Bouguer Plate:* Simply Bouguer reduction was calculated without the effect of the Earth's curvature due to the limited area of interest (William J. Hinze 2005) using the following formula

$$\delta gBC = 2\pi G\sigma H \quad (4)$$

where G=6.674x10^{11}Nm^2/kg^2 the universal gravitational constant
σ is the mean density of the rock material of the plate (in gr/cm^3)
H is the orthometric height above sea level

4) *Terrain Correction (TC)*: The calculation of TC for onshore measurements based on a four-step process. The first step defines a detailed correction using the 5 m grid from HMGS and the elevation of the station at a rectangular 10km x10km around each point. In the second step the 30m dem for a rectangular of 100km x100km around each point was used. Likewise at next two steps 50m and 100m dem was used at rectangular 200kmx200km and 400km x400km respectively around each point. Calculated TC are the addition of the values of the steps above. Calculation are based on following formula (Katsabalos & Tziavos,1991):

$$\mathrm{tc}(\mathrm{x_p}, \mathrm{y_p}) = \frac{1}{2}\mathrm{G}\rho \iint_S \frac{[\mathrm{h(x,y)}-\mathrm{h(x_P,y_P)}]^2}{\mathrm{l^3(x_P-x,y_P-y)}} \mathrm{dx\,dy} \quad (5)$$

Where tc terrain correction at the point P$(\mathrm{x_p}, \mathrm{y_p})$ with planar coordinates
G=6.6742*10-11 m3kg-1sec-2 worldwide constand
ρ=2670kgm-3 or ρ=1027 kgm-3 mean density of land and sea respectively
l the distance from any point (x,y) to point P
S the surface integral (10km x10km, 100kmx100km, 200km x200km or 400km x400km) from station P.

For the above calculations two separate scripts were written in python (version 2.7): topcor5_30.py and topcor200.py (Papadopoulos 2017) and the results presented at table 2:

Table 2. Calculated TC

Kind of points	Statistics (mgal)			
	Avg	min	max	sdv
Triangulation points	4.268	0.044	28.193	3.334
Gravity Database	3.89	0.00	35.20	2.99

B. *Gravity Anomalies*

According to the above reduction schema, Gravity anomalies can be calculated after the equations:

Free Air Anomaly (DgFA)= Gads-Gmod-δgFA (6)

Simple Bouguer Gravity Anomaly (SBG)= FAA- δgBC (7)

Complete Bouguer Gravity Anomaly (CBG)=SBG+TC (8)

C. *Geoid Calculation*

In order to calculate geoid undulation at the selected area a Geoid model has been computed, using the Least Square Collocation (LSC) method and remove compute restore (RCR) technique. The long wavelength contribution of the gravity field has been modelled by an Earth gravitational model obtained from *EIGEN6C4* from degree 0 to 2190. Owing to the roughness of the topography in some areas, terrain effects have been computed as mentioned above. The RCR technique for calculating the geoid model can be divided in three distinct stages:

1) ***Remove*** of the long and short wavelength component of the free air gravity anomaly. The said component is estimated by the gravity anomaly (ΔgGM) using the global geopotential models and terrain correction (DgTC). This process yields the residual anomaly (Δg RES):

$$Dgres = DgFA - DgGM - DgTC \quad (9)$$

where DgFA is the Free Air Anomaly
DgGM is free air model obtained by EIGEN6C4 and computed above.
DgTC is terrain effect calculated as described above

2) ***Compute*** the residual co-geoid model (Nres) using Dgres; the co-geoid model for the long wavelength components (Ngm) using the global geopotential models; and the primary indirect effect of topography (Nind), which is the vertical distance between the geoid and co-geoid:

$$Nres = Cs'lb \qquad (10)$$

Where

$$Cs'l = [CP1(N\Delta g)...CPi(N\Delta g)\ CP1(NN)...CPj(NN)] \qquad (11)$$

is the matrix of covariances for a grid of 15'' around each point and

$$b = (Css + Cnn)\text{-}1L \qquad (12)$$

is the least square prediction value,
where

$$Css = \begin{array}{c} C\Delta g\Delta g \mid C\Delta gN \\ \text{- - - - - - - -} \\ CN\Delta g \mid CNN \end{array} \qquad (13)$$

$$Cnn=0 \qquad (14)$$

because it is supposed that observations have no error, and

$$L = [\Delta g1\ \Delta g2\ ...\ \Delta gi\ N1\ N2\ ...\ Nj]T \qquad (15)$$

is the observation matrix.

*Indirect effec*t in planar approximation computed using the following formula (Wichiencharoen, 1982):

$$N_{ind} = -\frac{\pi G\rho h_p^2}{\gamma} - \frac{G\rho}{6\gamma}\iint_E \frac{h^3-h_p^3}{r_0^3}\,dx\,dy \qquad (16)$$

Where $G=6.674x10^{11} Nm^2/kg^2$ the universal gravitational constant

ρ is the mean density of the rock material of the plate (in gr/cm^3)

hp is the orthometric height at the computed point

h is the orthometric height at every point of computation

E is the planar area of the computation and

r_0 is the planar distance of the computed point and every point of computation

3) *Restore* above calculated values of NGM, Nres and Nind for the estimation of the geoid model (N)

$$N = Nres + NGM + Nind \qquad (17)$$

where NGM is the normal height obtained by EIGEN6C4

Nind is the primary indirect effect of topography.

Nres the computed co geoid.

D. *Geoid Model*

There was produced two different geoid models. One geoid model was produced using every available data described above. So measured geoid undulations at every measured point are used in LSC method Geoid height covariance function and cross covariance function of geoid height and gravity anomalies calculated from data and produce a Hybrid Ortho Biased Geoid model (hyb_OB).

Second geoid model was produced strictly from gravity anomalies and produced a Hybrid Ortho Free Geoid model (hyb_OF).

IV. RESULTS

A. *Comparison of two geoid models with GPS/levelling points*

Precise levelling measurements carried out from HGMS from 70s to early 90s at hole territory of Greece. More recently GPS measurements were obtained from HGMS and Hellenic Cadastre at selected triangulation points all over the country. Geoid undulation comparison from these datasets and the predicted values of the geoid models are presented at table 3:

Table 3. Statistics of two geoid model using 923 control points.

	ORTHO BIASED	ORTHO FREE
MEAN	0.008	0.203
MIN	-0.098	-1.738
MAX	0.102	1.058
SDV	0.048	0.688

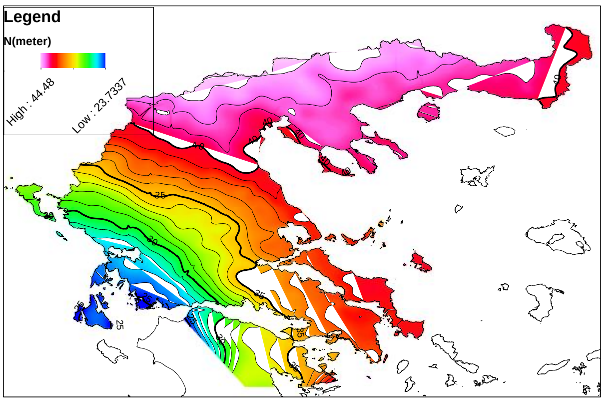

Figure 2. Hybrid Ortho Biased Geoid Model, which produced with LSC using measured geoid undulation N at triangulation points and gravity anomalies. Kriging method of interpolation in spherical approximation was used, with pixel size of 0.004583°~300m and contour interval 1m.

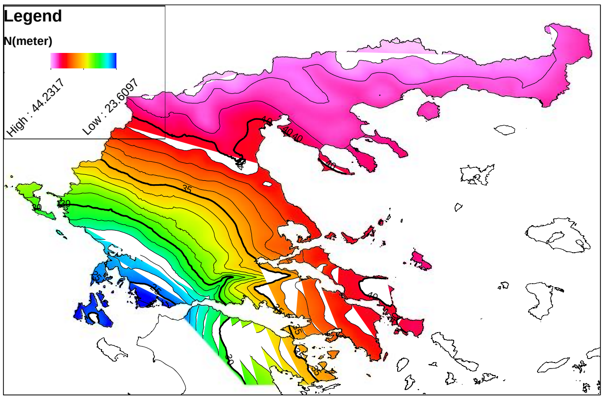

Figure 3. Hybrid Ortho Free Geoid Model. which produced with LSC using only gravity anomalies. Kriging method of interpolation in spherical approximation was used, with pixel size of .004583°~300m and contour interval 1m.

B. *Fitting GPS/Levelling geoid undulation to geoid*

The computed differences at control points reflect datum inconsistencies between the available height data, long-wavelength geoid errors, and GPS and leveling errors included in the ellipsoidal and orthometric heights. In order to minimize these deviations, we used a four- parameter transformation model. The four-parameter model is the most commonly used in such adjustments and is given by the following formula [Heiskanen and Moritz 1967, Sideris 1992]:

$$N_{GPSLev} - N_{grav} = h - H - N_{grav} = x0 + x1\cos\varphi\cos\lambda + x2\cos\varphi\sin\lambda + x3\sin\varphi \qquad (18)$$

where the parameters x0, x1, x2 and x3 are calculated by a least square adjustment of measured values at 1049 triangulation points in the area.

After fitting the surface new geoid model was produced Fitted Ortho Biased and Fitted Ortho Free Geoid Model. and the results at the control points presented at table 4:

Table 4. Statistics of two geoid model after fitting with GPS/LEVELLING, using 923 control points.

	FITTED ORTHO BIASED	FITTED ORTHO FREE
MEAN	0.006	0.020
MIN	-0.100	-0.934
MAX	0.100	0.703
SDV	0.048	0.242

From the results given in table 3 and 4, it can be concluded that the four parameter model affect only ortho free geoid model. Ortho biased geoid model has corrected the differences from the height system used by ggm compared to the national height system, using measured N at its solution routine.

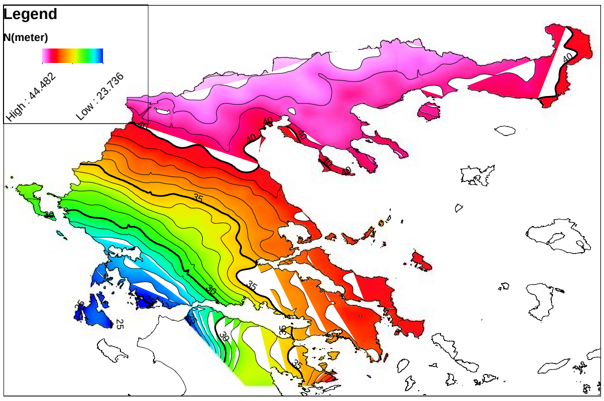

Figure 4. Hybrid Ortho Biased Geoid Model fitted with GPS/levelling control points using measured geoid undulation at points. Kriging method of interpolation in spherical approximation was used, with pixel size of 0.004583°~300m and contour interval 1m.

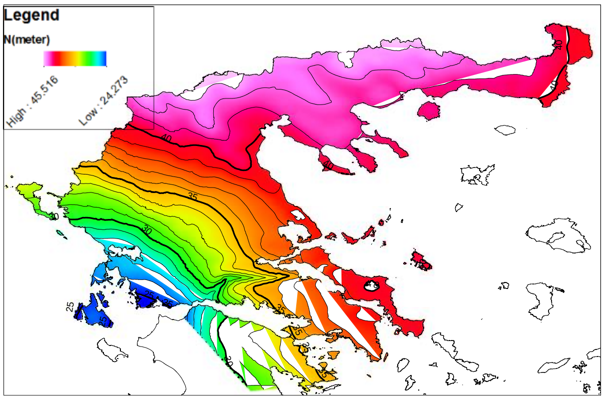

Figure 5. Hybrid Ortho Free Geoid Model fitted with GPS/levelling control points using measured geoid undulation at points. Kriging method of interpolation in spherical approximation was used, with pixel size of 0.004583°~300m and contour interval 1m.

V. CONCLUSIONS

It is obvious that after GPS techniques have become wide spread in geodetic purposes, geoid model determination for especially use in practical applications of geodesy has an increased importance.

Different data compilation schemes, ortho free and ortho biased models have been applied in the classical solution of RCR technique using LSC approach in one of the most rugged areas of Europe. Ortho biased model present the most accuracy gravity field for this area while ortho free model present most rough gravity field. The GPS/levelling fitting of the model does not improve the statistics of the ortho biased model but does improve that of the ortho free model. There is large difference in the statistics between ortho free model and fitted models with GPS-levelling ortho free Model. These large biases possibly indicate that the reference gravity field from EIGEN6C4 geopotential model does not agree exactly with the topography in this area.

Indirect effect on geoid undulation changes the geoid surface calculated in the whole region of Greece with a mean value of -1.3cm.

The major conclusion drawn from this study is that including geoid undulation at points in LSC method, compiled with gravity measurements, can produce extremely better results at geoid computation.

More studies regarding the improvement of absolute geoid for these models should be carried, including the whole Greek territory and much more GPS measurements, once it is find out that the methodology used is very accurate.

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