Statistical Verification of Real Estate Estimation Models[1]

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Key words: market value, multiplicative model, additive model, prediction

SUMMARY

The selection of a model describing the market variability of real estate values in relation to their characteristic attributes is the most important stage of the real estate market analysis. In the process of modelling the real estate values, additive or multiplicative functions can be used. A real estates database, containing transaction prices and attributes of real estates, may have additive as well as multiplicative qualities.

To choose a model describing the most precisely the variability of prices in a database, it is necessary to make the estimation of parameters of additive and multiplicative model and also a complete variance analysis. The verification of statistical hypotheses concerning the value difference of estimated parameters of these models will be the basis for a statistical inference allowing choosing a more reliable model. Parametric tests, constructed on *T*-Student's and *F*-Snedecor distribution quantiles, will be applied to the statistical inference.

On the basis of a selected model parameters and of valuated real estate attributes, the market value of a real estate will be determined with a complete precision evaluation. The real estate value, determined in such a way, being a prediction of the estimated model, can constitute the most probable market value of an analysed real estate.

In the final part of the paper, the practical numerical example of application and statistical verification of additive and multiplicative models in a database of real estates from Cracow area will be presented.

[1] The task is worked based on individual and statutory searches in Terrain Information Department AGH, Krakow, Poland

Statystyczna weryfikacja modeli wyceny nieruchomości

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Słowa kluczowe: wartość rynkowa, model addytywny, model multiplikatywny, predykcja

STRESZCZENIE

Dobór modelu do opisu zmienności rynkowej wartości nieruchomości względem wyróżnionych ich atrybutów stanowi najważniejszy etap analizy rynku nieruchomości. W procesie modelowania rynkowej wartości nieruchomości można stosować funkcje addytywne lub funkcje multiplikatywne. Rozważana baza informacji o nieruchomościach, zawierająca ceny transakcyjne i atrybuty nieruchomości, może posiadać cechy addytywne lub cechy multiplikatywne.

Aby wybrać model, który będzie najwierniej opisywał zmienność cen w bazie, trzeba dokonać estymacji parametrów modelu addytywnego oraz modelu multiplikatywnego wraz z pełną analizą wariancji. Weryfikacja hipotez statystycznych dotyczących różnicy wartości estymowanych parametrów tych modeli będzie podstawą do wnioskowania statystycznego, na podstawie którego można pojąć decyzję o wyborze modelu posiadającego lepszą wiarygodność. Do wnioskowania statystycznego będą wykorzystane testy parametryczne konstruowane na kwantylach rozkładu *T*-Studenta i rozkładu *F*-Snedecora.

Na podstawie parametrów wybranego modelu oraz atrybutów nieruchomości wycenianej będzie określana jej wartość rynkowa wraz z pełną oceną niedokładności. Tak określona wartość nieruchomości, stanowiąca predykcję estymowanego modelu, może stanowić najbardziej prawdopodobną wartość rynkową analizowanej nieruchomości.

W końcowej części pracy zostanie przedstawiony praktyczny przykład liczbowy zastosowania i statystycznej weryfikacji modelu addytywnego oraz modelu multiplikatywnego do baz nieruchomości z okolic Krakowa.

Statistical Verification of Real Estate Estimation Models[2]

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1. INTRODUCTION

The selection of a model describing the variability of real estate market value in relation to their characteristic attributes is the most important stage of the real estate market analysis. In the process of modelling the real estate market values, additive (1) or multiplicative (2) functions can be used in form:

$$w = a_0 + \sum_{k=1}^{m} g(X_k) \tag{1}$$

where:

- w – unit price or value of real estate,
- X_k – value of attribute k for real estate,
- g – function of real estate price - attribute k relation,
- a_0 – free term in the model (unit value of a real estate, for zero of all attributes).

$$w = a_0 \cdot a_1^{x_1} \cdot a_2^{x_2} \cdot \cdots a_m^{x_m} \tag{2}$$

where:

- w – unit price or value of a real estate,
- $x_1, x_2, \ldots, x_m$ – attributes of real estates,
- a_j – estimated model coefficients,
- a_0 – free term in the model (unit value of a real estate, for zero of all attributes).

The analyzed real estates database, containing transaction prices and attributes of real estates may have additive or multiplicative qualities. The selection a priori of an appropriate model form for estimating a real estate on the basis of gathered database is not possible. This report aims to show that even on markets apparently very resembling, the optimum form of the function used to model a real estate value can be different. Therefore, an estimation of model parameters for both types with a full analysis of variance as one of the elements of the model quality statistical analysis must be done. Suitable statistical tests verifying the significance of estimated model parameters and comparing them are necessary too.

2. ESTIMATION OF MODELS PARAMETERS

2.1. Estimation of the additive model

First of all, we will consider a special case of a system of equations in form (1), which is a linear multiple regression (linear non-only by parameters, but also by independent variables):

[2] The task is worked based on individual and statutory searches in Terrain Information Department AGH, Krakow, Poland

$$w = a_0 + \sum_{k=1}^{m} X_k \cdot a_k \tag{3}$$

The expression (3) may be written as a matrix:

$$[W] = [X] \cdot [a] \tag{4}$$

where:

$W = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$ – vector of dependent random variable (of a real estate value),

$X = \begin{bmatrix} 1 & x_{11} & \cdots & x_{1m} \\ 1 & x_{21} & \cdots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & \cdots & x_{nm} \end{bmatrix}$ – matrix containing ones and independent variables (attributes),

$a = \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_m \end{bmatrix}$ – vector of multiple linear regression coefficients.

The application of least squares method, i.e. the determination of the estimator $\hat{a}$ of a coefficient vector *a* such that:

$$\sum_{i=1}^{n} (c_i - w_i)^2 = \delta^T \delta = \min \tag{5}$$

where:

c_i – price observed for i^{th} real estate in the database,
w_i – model real estate value,
$\delta = C - W$ – vector of random remainders (differences between a price observed *C* and a value forecasted according to the model *W*),

in every case leads to a non-linear system of normal equations, which is solved using numerical iterative procedures.

Solving a generalized linear model means:

- determining an unbiased estimator of vector of unknowns:

$$\hat{a} = (X^T X)^{-1} \cdot X^T C \tag{6}$$

- determining an unbiased estimator of remainder variance (defining estimation inaccuracy of model parameters):

$$\hat{\sigma}_0^2 = \frac{C^T C - \hat{a}^T X^T C}{n - m - 1} \tag{7}$$

- determining a covariance matrix of vector of unknowns (model parameters):

$$Cov(\hat{a}) = \hat{\sigma}_0^2 \cdot (X^T X)^{-1} \tag{8}$$

- determining a covariance matrix of model values:

$$Cov(W) = \hat{\sigma}_0^2 \cdot X^T (X^T X)^{-1} X \tag{9}$$

The value of each regression coefficient is presented with significance level, which determines the reliability of the value found on the grounds of a test in relation to the whole population.

In the general structure of an additive model (1), we will find polynomials of different degrees as functions *g*. Thus, developing the model (1), we have:

$$c = a_0 + \sum_{k=1}^{m} (a_{k_1} \cdot X_k + a_{k_2} \cdot X_k^2 + ... + a_{k_{n_k}} \cdot X_k^{n_k}) \tag{10}$$

where:

m – number of attributes considered in the model,
n_k – degree of polynomial for k-th attribute.

At polynomial forms of the function *g*, the whole model maintains the linearity in relation to the parameters. Therefore, the estimation of the model parameters may be done the same way as in the case of multiple regression. The matrix *X* takes on the form:

$$X = \begin{bmatrix} 1 & x_{11} & x_{11}^2 & \cdots & x_{11}^{n_1} & x_{12} & x_{12}^2 & \cdots & x_{12}^{n_2} & \cdots & x_{1m} & x_{1m}^2 & \cdots & x_{1m}^{n_m} \\ 1 & x_{21} & x_{21}^2 & \cdots & x_{21}^{n_1} & x_{22} & x_{22}^2 & \cdots & x_{22}^{n_2} & \cdots & x_{2m} & x_{2m}^2 & \cdots & x_{2m}^{n_m} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & x_{n1}^2 & \cdots & x_{n1}^{n_1} & x_{2n} & x_{2n}^2 & \cdots & x_{2n}^{n_2} & \cdots & x_{nm} & x_{nm}^2 & \cdots & x_{nm}^{n_m} \end{bmatrix} \tag{11}$$

2.2. Estimation of Multiplicative Model

For the estimation of the coefficients a_j in model (2), the function (2) has to be brought to a linear form. For this purpose, we take the logarithms of both sides, using the natural logarithm, and we receive:

$$\ln w = \ln a_0 + x_1 \cdot \ln a_1 + x_2 \cdot \ln a_2 + ... + x_m \cdot \ln a_m \tag{12}$$

The system of equations (12) has the features of a probabilistic model, taking on in matrix notation the following form:

$$[\ln W] = [X] \cdot [\ln a] \tag{13}$$

where:

$[\ln W]$ – vector $[n \times 1]$, containing natural logarithms of database real estates prices,

$[X]$ – rectangular vertical matrix $[n \times (m+1)]$, containing ones and attributes values of real estates in database,

$[\ln a]$ – vector $[(m+1) \times 1]$, containing estimated values of logarithms of the model natural coefficients a_j,

where n is the number of considered real estates in the database and m is the number of considered attributes.

The system of equations (13) rewritten as a full matrix form becomes:

$$\begin{bmatrix} \ln w_1 \\ \ln w_2 \\ \ln w_3 \\ \dots \\ \ln w_n \end{bmatrix} = \begin{bmatrix} 1 & x_{11} & x_{12} & \dots & x_{1m} \\ 1 & x_{21} & x_{22} & \dots & x_{2m} \\ 1 & x_{31} & x_{32} & \dots & x_{3m} \\ \dots & \dots & \dots & \dots & \dots \\ 1 & x_{n1} & x_{n2} & \dots & x_{nm} \end{bmatrix} \cdot \begin{bmatrix} \ln a_0 \\ \ln a_1 \\ \ln a_2 \\ \dots \\ \ln a_m \end{bmatrix} \tag{14}$$

By applying the least squares weighing method and taking into account Gauss-Markow principle, we receive the following formulas for estimated parameters of a non-linear model:

$$\ln \hat{a} = \left(X^T X\right)^{-1} \cdot X^T \ln C \tag{15}$$

where:

C – vector of unit price or value of real estates in a given database

To determine a covariance matrix for estimated regression coefficients, we use the formula:

$$Cov(\ln \hat{a}) = \hat{\sigma}_0^2 \left(X^T X\right)^{-1} \tag{16}$$

where σ_0^2 is the variance of the multiplicative non-linear model estimation. Its estimator is given by the following expression:

$$\hat{\sigma}_0^2 = \frac{(\ln C)^T \cdot (\ln C) - (\ln \hat{a})^T \cdot X^T \cdot (\ln C)}{n - m - 1} \tag{17}$$

The elements on the main matrix diagonal (16) are the squares of natural logarithms standard deviations for particular coefficients of a non-linear model (2), i.e.:

$\sigma^2(\ln a_0),\ \sigma^2(\ln a_1),\ \sigma^2(\ln a_2), \dots, \sigma^2(\ln a_m)$.

3. EXAMINING THE SIGNIFICANCE OF MODEL COEFFICIENTS

Within the framework of the statistical verification of estimated models, we examine the significance of the parameter system in each model and the significance of every particular parameter. However, the basic indicator determining the quality of matching a model with data is the square of curvilinear correlation coefficient R^2. If $W = F(X,a)$ is a second-degree hypersurface determined on the basis of a random test (X,C),

where:

X – attributes of real estates in a given database,

C – price or value of real estates in a given database,
then R^2 is defined as:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}[c_i - w_i]^2}{\sum_{i=1}^{n}[c_i - E(c)]^2} = \frac{\sum_{i=1}^{n}[w_i - E(c)]^2}{\sum_{i=1}^{n}[c_i - E(c)]^2} \Leftrightarrow R^2 = 1 - \frac{NSK}{CSK} = \frac{WSK}{CSK} \tag{18}$$

where:

E(c) – mean value of dependent variable observed values (prices),

$\sum_{i=1}^{n}[c_i - E(c)]^2$ – total dispersion of a dependent variable in relation to its mean value (*CSK*),

$\sum_{i=1}^{n}[c_i - w_i]^2 = \sum_{i=1}^{n}\delta^2$ – sum of squares of deviations between the values of a dependent variable from the test and its model values, expressing the part unexplained by a non-linear regression model (*NSK*),

$\sum_{i=1}^{n}[w_i - E(c)]^2$ – sum of squares of deviations between the model values of a dependent variable and the mean value of its observed values, expressing the part explained by a non-linear regression model (*WSK*).

3.1. Examining the Significance of Model System Coefficients

Verification of a parameters system significance is based on Fisher-Snedecor's statistics, using the following hypothesis H_0: $\sum_{j=0}^{m} a_j^2 = 0$ against the alternative hypothesis H_1: $\sum_{j=0}^{m} a_j^2 \neq 0$. Statistic form in this test:

$$F = \frac{R^2}{1 - R^2} \cdot \frac{n - m - 1}{m} \tag{19}$$

which, if the null hypothesis is true, has a *F*-Snedecor's distribution with (*m*, *n*-*m*-1) degrees of freedom.

3.2. Examining the Significance of Particular Model Coefficients

Verification of particular regression coefficients significance is based on *T*-Student statistics, under the null hypothesis H_0: $a_j = 0$ against the alternative hypothesis H_1: $a_j \neq 0$. Statistic form in this test:

$$T = \frac{\hat{a}_j}{\sigma(\hat{a}_j)} \tag{20}$$

which, if the null hypothesis is true, has a *T*-Student distribution with (*n*-*m*-1) degrees of freedom.

If for any of explaining variables, the statistical test does not demonstrate reasons for rejecting the null hypothesis, we estiminate this variable from the model and we reestimate the parameters.

4. DETERMINATION OF REAL ESTATE PREDICTED VALUES

Basing on the parameters of a selected model and on the attributes of an estimated real estate, we determine its value with full accuracy analysis. The analyzed real estate value determined in such a way, being a prediction of estimated model, can be its most probable market value.

4.1. Prediction of a Market Value on the Basis of an Additive Model

On the basis of the model parameters, the prediction of values of real estates selected from a given market, is performed according to the following formula:

$$w = \begin{bmatrix} 1 & x_1 & x_2 & \dots & x_m \end{bmatrix} \cdot \hat{a} \tag{21}$$

where:

$\begin{bmatrix} 1 & x_1 & x_2 & \dots & x_m \end{bmatrix}$ – vector of values of estimated real estate attributes,

$\hat{a}$ – vector of estimated model parameters.

The accuracy of such a prediction is evaluated by its variance expressed by formula:

$$\sigma^2(w) = \begin{bmatrix} 1 & x_1 & x_2 & \dots & x_m \end{bmatrix} \cdot Cov(\hat{a}) \cdot \begin{bmatrix} 1 & x_1 & x_2 & \dots & x_m \end{bmatrix}^T \tag{22}$$

4.2. Prediction of a Market Value on the Basis of a Multiplicative Model

The prediction of an arbitrary real estate unit market value w is determined by substituting to the evaluated model in the form (12) the values of the attributes x_k. By converting this, we receive the forecast value for the analysed real estate:

$$w = \exp\left(\ln a_0 + x_1 \cdot \ln a_1 + x_2 \cdot \ln a_2 + \dots + x_m \cdot \ln a_m\right) \tag{23}$$

The standard deviation of the forecast price natural logarithm is determined using the covariance matrix (16) for the coefficients of the linearizated model using the expression:

$$\sigma^2(\ln w) = \begin{bmatrix} 1 & x_1 & x_2 & \dots & x_m \end{bmatrix} \cdot Cov(\ln \hat{a}) \cdot \begin{bmatrix} 1 & x_1 & x_2 & \dots & x_m \end{bmatrix}^T \tag{24}$$

The standard deviation of forecasted market value, accordingly to the law of error propagation is calculated by formula:

$$\sigma(w) = w \cdot \sigma(\ln w) \tag{25}$$

Analysing the variance, we can state that the forecast price (value) of the analysed real estate, at confidence level (1-α)=0.95, should be contained in the following interval:

$$w \pm t(0{,}975; n - m - 1) \cdot \sigma(w) \tag{26}$$

with $t(0.975; n\text{-}m\text{-}1)$ denoting an appropriate quantile of T-Student distribution.

5. EXAMPLE

In the tables 1 and 2 below, we present some of model estimation results in additive (linear in consideration of independent variables and parameters or in relation to the parameters only) and multiplicative form. Models were tested on two local markets of real estates of the same type (dwellings). Acquired information on transactions concern two big quarters of Cracow, diversified in respect of factors shaping the prices of real estates.

Last two lines in each table, include the test results of verification of parameter system significance.

Estimation results of model in additive form:

Table 1. Results of additive model estimation

	multiple regression		linear model in relation to the parameters	
	Quarter A	Quarter B	Quarter A	Quarter B
n	91	77	91	77
u	14	14	29	40
σ_0^2	0,17	0,08	0,15	0,08
σ_0	0,42	0,28	0,39	0,28
R^2	0,68	0,85	0,78	0,91
s	0,57	0,40	0,10	0,07
$F(\alpha, u-1, n-u)$	1,85	1,86	1,66	1,72
F_{cal}	12,68	27,52	7,68	10,44

where:

n – number of real estates in a database,
u – number of model estimated parameters,
σ_0^2 – model remainder variance,
σ_0 – estimation standard error,
R^2 – coefficient of determination,
s – fraction of model coefficients statistically significant in their total number,
$F(\alpha, u-1, n-u)$ – critical value of Fisher-Snedecor's test, verifying the significance of a system of coefficients, at significance level $\alpha = 0{,}05$,
F_{cal} – calculated value of a test function (statistics).

Estimation results of model in multiplicative form:

Table 2. Results of multiplicative model estimation

	Quarter A	Quarter B
n	91	77
u	14	15
σ_0^2	0,02	0,01
σ_0	0,14	0,07
R^2	0,68	0,84
s	0,57	0,40
$F(\alpha, u-1, n-u)$	1,85	1,86
F_{cal}	12,49	23,63

When we eliminate from the model variables which are not statistically significant, the new estimation of parameters is made and the model is verified. After this procedure all of parameters in each model are significant (s=1,00). The results of the analysis of qualities of reduced models are presented in tables 3 and 4.

In the case of a model in additive complex form for the quarter A, where the coefficient of matching is very low R^2=0,26, the analysis of accuracy as well as the continuation of model verification were abandoned. This model was found unfit to predict the market value of a real estate.

Estimation results of model in additive form:

Table 3. Results of estimation of additive reduced models

	multiple regression		linear model in relation to the parameters	
	Quarter A	Quarter B	Quarter A	Quarter B
n	91	77	91	77
u	8	6		2
σ_0^2	0,17	0,22		0,22
σ_0	0,42	0,47		0,47
R^2	0,65	0,55	**0,26**	0,74
s	1,00	1,00		1,00
$F(\alpha, u-1, n-u)$	2,13	2,35		3,97
F_{cal}	21,86	17,13		211,64

Estimation results of model in multiplicative form:

Table 4. Results of estimation of multiplicative reduced model

	Quarter A	Quarter B
n	91	77
u	8	6
σ_0^2	0,02	0,01
σ_0	0,15	0,07
R^2	0,64	0,82
s	1,00	1,00
$F(\alpha, u-1, n-u)$	2,13	2,35
F_{cal}	21,53	23,63

Comparing coefficients of determination R^2 obtained for models in the quarter *A* before and after reducing parameters in models (0,68 and 0,65 in additive simple model; 0,68 and 0,64 in multiplicative model), we can notice that the reduction of number of variables is negligible for quality of model.

On the ground of obtained results, we can see that the additive form of the model illustrates better the variability of prices in relation to the qualities essential for it in the quarter *A*. Whereas, in the quarter *B* the multiplicative model seems to be better matched after the elimination of parameters statistically insignificant from the models.

In all models, before and after simplification, the system of estimated parameters shows the statistical significance. It is however noticeable that the percentage of statistically significant parameters in the model decreases considerably when the number of parameters increases (Table 1). Though, it does not cause a proportional increase of model matching. Then, complicating the model by multiplication of parameters would not necessary improve its overall quality (example of quarter *B*, where the additive model in simple form at the matching R^2=0,85 had 40% of parameters statistically significant, while the complex additive model, where the matching was only 5% better (R^2=0,91) has only 7% of significant parameters).

6. SUMMARY

Recapitulating, we can say that the vicinity of local real estate markets, like in case of quarters of the same town, as well as the consideration of real estates of the same type, are not sufficient to presume that an equal form of the estimation model will be proper.

In the analyzed example, for the quarter *A*, finally, after the elimination of statistically insignificant parameters (variables), both forms of estimation model can be applied. However, for the quarter *B*, the multiplicative model seems to match better the market data, giving a lower estimation standard error than in case of additive model.

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