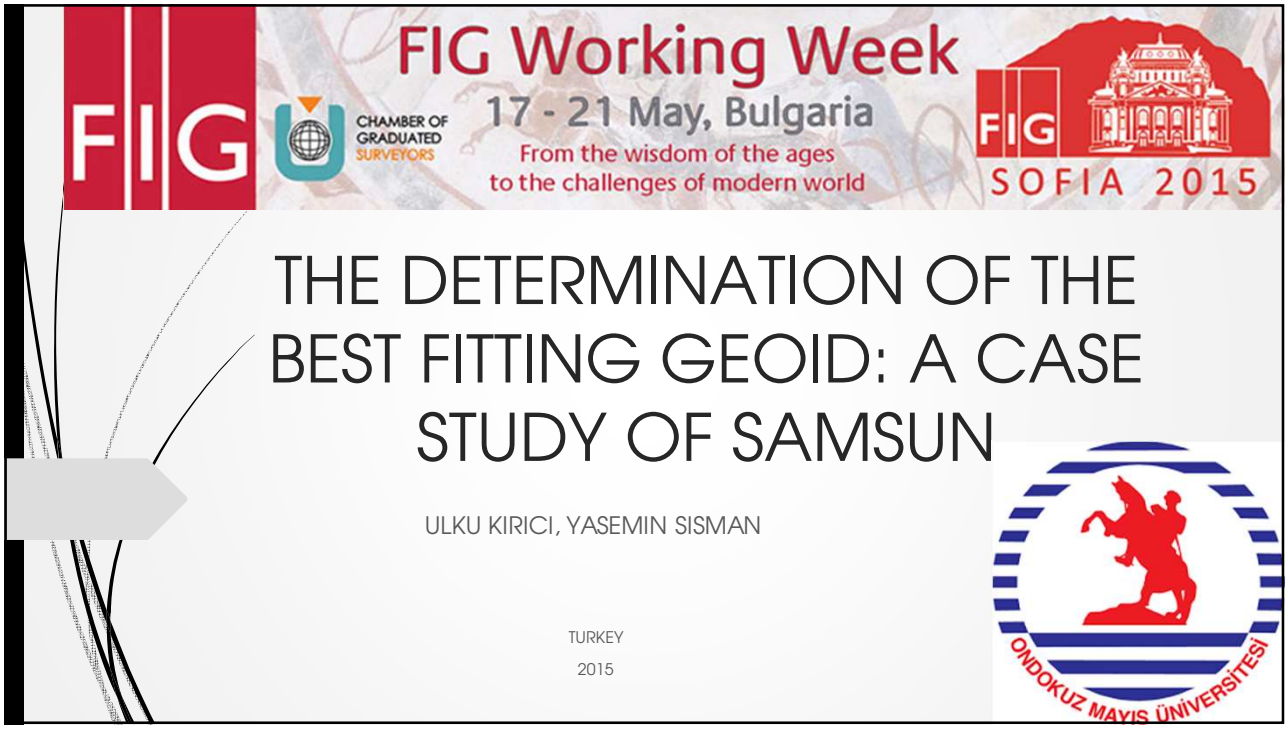


THE DETERMINATION OF THE BEST FITTING GEOID: A CASE STUDY OF SAMSUN

ULKU KIRICI, YASEMIN SISMAN

TURKEY

2015

ONDOKUZ MAYIS ÜNİVERSİTESİ

1. INTRODUCTION

- The geoid determination is the most important problem for scientist interested in the earth. There are a lot of areas interested in geoid like geodesy, geophysics, geography etc. The geoid called the surface closed the average sea surface and formed by the combination of the points have got zero potential value. The geoid is a complex surface and it is not easy defined as mathematically. In the geodesy the measurements on the physical earth, but the calculation of measurements is done on the reference surface. Thus, the difference between the reference ellipsoid was called geoid ondulation. The geoid determination methods had been developed to obtained geoid ondulation values.

The Geoid Determination Methods Can Be Given Following

1. The astro-geodetic method
2. The gravity values
   a. The stokes function
   b. The Fourier transformation
3. Geoid determination according to numerical density method
4. The geopotential approach
5. The combined methods (remove - restored)
6. The GPS/levelling methods
   a. The polynomials method
   b. The fuzzy logic method
   c. The Artificial neural network
   d. Etc.

The astro-geodetic method is first method using geoid determination. In the early 1970s, the geopotential models have been developed for geoid determination. 1980s the fast Fourier transformation has been used to obtained gravity data. In 1990s the satellites geodesy was started to spread rapidly. Then, the combination of the GPS and levelling data began to be used for geoid determination.

- The most commonly used geoid determination method is the polynomial method. This method is defined a polynomial function by using the point 3D coordinates and can be applied regionally. In this study, it is try to the best suitable geoid for Samsun were determined using the Samsun Levelling and GPS Network data using polynomial method. The data of study includes the 478 point coordinates.

2. GEOID DETERMINATION WITH POLYNOMIALS

- This method is the most widely used surface fitting procedure. A surface is obtained from GPS ellipsoidal and orthometric heights of points. The function of surface is determined with basic definition of orthogonal polynomials:

$$N_{(x,y)} = \sum_{i=0}^{m} \sum_{\substack{j=0 \\ j=k-i}}^{k} a_{ij} x^i y^j$$

- where (*x, y*) is the position coordinates of points, *aij* the constants of the polynomial and *m* the order of the chosen polynomial. 2nd order polynomial equation can be written for the polynomial:

$$N_{(x,y)} = a_{00} + a_{01}y + a_{10}x + a_{02}y^2 + a_{11}xy + a_{20}x^2$$

- The measurement and unknown numbers are equal to the point and constants number. If the measurement number (n) is bigger than the unknown number (u), the solution must be realized by using adjustment procedure. When the designed according to indirect measurement adjustment mathematical model, the following equations are obtained:

$$V = AX - \ell \qquad P_{\ell\ell} = Q_{\ell\ell}^{-1} \qquad X = \begin{bmatrix} a_{00} \\ a_{01} \\ a_{10} \\ a_{02} \\ a_{11} \\ a_{20} \end{bmatrix} \qquad A = \begin{bmatrix} 1 & y_1 & x_1 & y_1^2 & x_1 y_1 & x_1^2 \\ 1 & y_2 & x_2 & y_2^2 & x_2 y_2 & x_2^2 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 1 & y_n & x_n & y_n^2 & x_n y_n & x_n^2 \end{bmatrix} \qquad \ell = \begin{bmatrix} N_1 \\ N_2 \\ . \\ . \\ N_n \end{bmatrix}$$

- This model can be solved by objective function of the least square adjustment method. The unknown parameters are obtained following equation

$$X = \left(A^T P A\right)^{-1} A^T P \ell$$

2.1. Outlier Detection

- The measurement group has got the outliers inevitably. These outliers can adversely affect the adjustment. Therefore, the outlier detection test must be done to determine the outliers measurements.
- The outlier detection test is realized according to hypothesis is used for outlier detection. The test size is calculated by using the residuals of measurements and their standard deviation .

$$T_i = \frac{|V_i|}{m_{vivi}} = \frac{|V_i|}{m_0 \sqrt{Q_{vivi}}}$$

- This test value is compared with the q table values.

$$q = T_{f,1-\alpha/2} = \sqrt{\frac{f\, t2f-1,1-\alpha/2}{f-1+t2f-1,1-\alpha/2}}$$

- If, $T_i > T_{f,1-\alpha/2}$ this measurement is accepted as outlier measurements. The measurement has got a biggest value as outlier is removed the measurement group, then this procedure repeated until there are no outlier measurement.

2.2.The Determination of The Best Suitable Polynomial Order

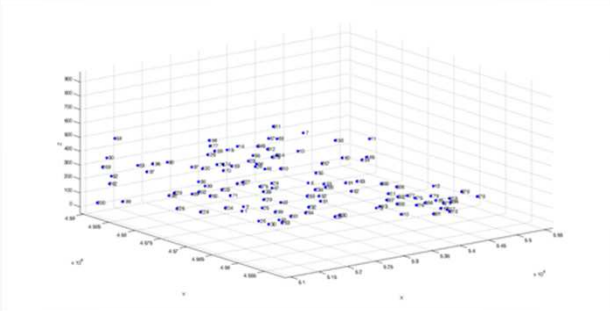
- The significance test and the changing of a posteriori variance can be used to determine the best suitable polynomial order. The significance test of unknown parameters is realized by using the unknown value and their standard deviation . For this procedure, the hypothesis test is established following.
- H_o : E(xi)= 0 zero hypothesis
- H_s : E(xi) ≠ 0 alternatively hypothesis
- Then, the test value is calculated.

$$T_i = \frac{|x_i|}{m_{xi}}$$

- The test value is compared to the $q=t_{f,1-\alpha/2}$ table value. If Ti is less than q , it is accepted zero hypothesis. This unknown parameter is not significant statistically.
- The best suitable of the polynomial order can be determined by using changing of a posteriori variance. The order of the polynomial increases, the variance is reduced. This reduction will stop at one step. This step is determined the suitable polynomial order.

3. NUMERICAL APPLICATION

- The data of Samsun triangulation network was used for application. This data includes the ellipsoidal coordinate of 478 points as (X,Y,h) and orthometric height values as (H).



- The polynomial function was obtained from 1 to 6 order by using adjustment solution according to least square method. The outlier detection was realized in all solution until there were not outlier measurements in data. The outlier test results were given in Table .

1st order		2nd order		3rd order		4th order		5th order		6 orders	
Outlier Point		Outlier Point		Outlier Point		Outlier Point		Outlier Point		Outlier Point	
Num	NN	Num	NN	Num	NN	Num	NN	Num	NN	Num	NN
19	418, 417, 427, 428, 426, 187, 167, 429, 419, 178, 12, 186, 449, 430, 185, 450, 431, 179, 448	7	188, 418, 427, 417, 428, 426, 7	4	188, 378, 7, 25	9	188, 378, 25, 7, 10, 100, 274, 261, 275	19	188, 378, 25, 7, 10, 478, 100, 274, 261, 68, 275, 11, 49, 51, 412, 386, 380, 13, 6	16	188, 378, 25, 10, 7, 100, 274, 261, 68, 11, 275, 6, 49, 51, 13, 300

- The significance test was performed on the unknown parameters after outlier detection test. It was found that only the first term was significant for each polynomial order. The changing of a posteriori variance was obtained using consistent measure group for each polynomial.

The order of the polynomial	Standard deviation values (cm.)
1st order	22.428
2nd order	19.687
3rd order	9.643
4th order	4.020
5th order	2.996
6th orders	2.789

- It was found that the changing of posteriori variance after 6th order term was decreased. In this case, it was decided that the best suitable value of polynomial function was obtained 6th order.

4. CONCLUSION

- The polynomial geoid determination is the most widely used method. In this study the best suitable geoid was determined for the Samsun triangulation network.
- For this application 478 points were used. The first step of application was to determine the outlier measurements group of data by using the outliers test. Most outlier points have been found in 1 and 5 order polynomial function. It was found in at least outlier points in the 3 degree polynomial function.
- Then, the significance test of unknown parameters and the changing of a posteriori variance were made to determine the best suitable order of polynomial function. The significance testing was found the only first term was significant in all polynomial function.
- The result of this procedure it can be said that the zero-order polynomial best fit polynomials function. The changing of a posteriori variance was not observed from 6th to 7th order polynomial function. In this case, it is decided that the best suitable geoid determination function was the 6th order polynomial function for the Samsun triangulation network.

THANK YOU FOR YOUR LISTENING…