The Features of the Coordinate Transformation from the Geodetic System WGS84 with the Mercator Projection for Low Latitudes Conditions

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This work presents the computation procedures of a rectangular plan coordinates using data obtained through satellite observations when creating geodetic networks. The peculiarity of these works is in the coordinate conversion to the Mercator projection. The selection of the projection of coordinates is necessary for each condition, which significantly differs from a place to another in different localities on the surface of the Earth. When using the technology of global navigation satellite system, this task is relevant for any point (area) of the Earth due to a fundamental different approach in determining the coordinates. The fact that satellites determination is more precise than the ground coordination methods (i.e. triangulation). In addition, the conversion to the zonal coordinate system is associated with errors; the value at present can prove to be completely critical. Moreover, the conversion into zonal coordinate system is associated with errors, the value of which at present can prove to be completely curtail. The proposed methodology was conducted over the Lebanese territory. The expediency of using the Mercator projection in the topographic and geodetic works production at low latitudes is shown numerically on the basis of model calculations. To convert the coordinates from the geodetic system with the Mercator projection, a programming algorithm which is widely used in surveying has been adopted. Results showed that that the difference between coordinates in the proposed transformation method is within an order of 0.04 m. Accordingly, the modified algorithm can be successfully used to convert geodetic coordinates to plane rectangular coordinates.

Keywords: Geodetic network, scale factor, Algorithms, Lebanon

INTRODUCTION

Usually, setting designs of any engineering work requires the utilization of rectangular plane coordinate system. Currently, its preparation is conveniently and effectively accomplished by the recalculation of geodetic coordinates. This is due to the rapid introduction into practice of geodetic works of satellite definitions. The coordinates of the points are obtained in the world geodetic coordinate system WGS-84 (GPS). Further, these coordinates are converted into a plane rectangular coordinate system. Thus, the general principle of the modern method of establishment of geodetic basic for the performance of construction works consists in determining geodetic coordinates using satellite receivers and transforming them into plane rectangular. In low latitudes, it is highly advisable to use the transverse Mercator projection [9].

METHODOLOGY

There are a number of algorithms used to transform from one coordinates system into another [1–8]. As part of these studies, the algorithm adopted in countries located in low latitudes [4-8] is considered, as well as the modified algorithm presented in [2, 3] for converting to the Mercator projection according to geodetic coordinates (ellipsoid WGS-84).

Briefly, we should note that in the system of plane rectangular coordinates in the Mercator projection, the X axis represents the equator is directed to the east, and the Y axis is the axial Meridian and directed to toward north. In this projection, the earth's surface is divided into three - and six-degree zones (discussed the six-degree zone). In the Mercator projection the first zone is that in which the axial meridian has a longitude of 177° (west).

Thus, for conditions of low latitudes when using global navigation satellite systems (GNSS) positioning needs a convenient, straightforward and effective method for converting geodetic coordinates to plane rectangular for specific engineering works. The task was to study the existing algorithms of coordinate transformation and to choose an effective conversion method. Below we show the well-known algorithms: given in [2, 3, 4–8], which can be called traditional. We Notice that the traditional method involves rather bulky formulas that are not convenient for automated calculations. The modified algorithm for low-latitude conditions is represented by more suitable for programming formulas.

Below, presented is the sequence of calculations using the traditional algorithm for low latitude conditions [4-8].

After determining the geodetic ellipsoidal coordinates (B, L) using the GNSS technology (coordinates on the WGS-84 ellipsoid) calculated are the plane rectangular coordinates (x, y) of the desired points in the Mercator projection.

1. Determine the zone number *n* by the longitude of the desired point:

$$n = TRUNC\left(\frac{L + L_{W0}}{6}\right), \quad (1)$$

Where, TRUNC – Truncates a number to an integer by removing the fractional part of the number; L – longitude of the point to be determined; L_{W0} – the longitude of the western boundary of the zone zero for the Mercator projection, $L_{W0} = 186^\circ$.

2. Determine the longitude of the Central Meridian L_0 of the zone n and the difference in longitude l at this point:

$$L_0 = 6 \cdot n - L_{C0}, \quad (2)$$

$$l = L - L_0, \quad (3)$$

Where, L_{C0} – the longitude of the Central Meridian of the zero zone for the Mercator projection, $L_{C0} = 183^\circ$.

3. Determine the eccentricity of the ellipsoid WGS-84

$$e = \sqrt{1 - \frac{b^2}{a^2}}, \quad (4)$$

Where, *a, b* – respectively, the major and the minor semi-axes of the WGS-84 ellipsoid, a = 6378137 m, b = 6356752.3142 m.

4. Determine the Radius of curvature on the plane of the prime vertical:

$$N = \frac{a}{\sqrt{1 - e^2 \cdot (\sin B)^2}}, \qquad (5)$$

Where, B – the latitude of the determined point

5. Determine the True distance along central meridian from the equator to the latitude (across from the point):

$$X = a \cdot \left[G_0 \cdot B - G_1 \cdot \sin(2B) + G_2 \cdot \sin(4B) - G_3 \cdot \sin(6B)\right], \qquad (6)$$

Where, G_0, G_1, G_2, G_3 – coefficients determined by formulas:

$$\left.\begin{aligned} G_0 &= 1 - \frac{1}{4}e^2 - \frac{3}{64}e^4 - \frac{5}{256}e^6 \\ G_1 &= \frac{3}{8}e^2 + \frac{3}{32}e^4 + \frac{45}{1024}e^6 \\ G_2 &= \frac{15}{256}e^4 + \frac{45}{1024}e^6 \\ G_3 &= \frac{35}{3072}e^6 \end{aligned}\right\}, \qquad (7)$$

6. Determine plane rectangular coordinates (x, y) in the Mercator projection:

$$x = F_e + A \cdot N \cdot K_0 \cdot \left[\begin{aligned} & 1 + \frac{A^2}{6} \cdot (1 - T + C) + \\ & \frac{A^4}{120} \cdot \left(5 - 18 \cdot T + T^2 + 72 \cdot C - 58 \cdot \left(\frac{e^2}{1 - e^2}\right)\right) \end{aligned}\right], \qquad (8)$$

$$y = K_0 \cdot \left[X + \frac{l^2 \cdot N \cdot \cos B \cdot \sin B}{2} \cdot \left[\begin{aligned} & 1 + \frac{A^2}{12} \cdot \left(5 - T + 9 \cdot C + 4 \cdot C^2\right) + \\ & \frac{A^4}{360} \cdot \left(61 - 58 \cdot T + T^2 - 330 \cdot \left(\frac{e^2}{1 - e^2}\right)\right) \end{aligned}\right]\right], \qquad (9)$$

Where, F_e – false easting, equal 500000 m.

K_0 – scale factor at the central meridian, equal 0.9996;

Coefficients A, C, T determined by formulas:

$$\begin{bmatrix} A = l \cdot \cos B \\ C = \dfrac{e^2 \cdot (\cos B)^2}{1-e^2} \\ T = (tgB)^2 \end{bmatrix}, \tag{10}$$

Now formulas for the modified algorithm for low-latitude conditions [2, 3] shown in (table.1). Note the differences in formulas comparing with the traditional methods of coordinate transformation.

Table 1: Modified algorithm for low-latitude conditions
(Ellipsoid WGS-84)

$x = Fe + \dfrac{k_0 \cdot N}{\sqrt{V}} \cdot arth\left(\sqrt{V} \cdot \cos B \cdot \sin l\right)$
$y = Y + \Delta y$
$Y = \dfrac{\mathrm{Re} \cdot k_0}{\rho} arctg\left[\dfrac{\mathrm{Re} \cdot \sqrt{1-e^2}}{a} \cos\left(\dfrac{\cos B}{6}\right) tgB\right]$
$\Delta y = \dfrac{N}{\rho} \cdot \left[arctg\left(\dfrac{tgB}{\cos l}\right) - B\right] / t$
$\mathrm{Re} = \dfrac{3}{4}(a+b) - \dfrac{1}{2}\sqrt{a \cdot b}$
$q = \cos(0{,}003 \cdot \sin 2B)$
$N = \dfrac{a}{\sqrt{1-e^2(\sin B)^2}}$
$t = 1 + 0{,}00084 \cdot (\cos B)^4$
$V = \sqrt{1 + \left(\dfrac{e^2}{1-e^2}\right) \cdot (\cos B)^2}$

Re – Equivalent Earth Radius

q – Isometric latitude

t – Correction factor to ensure millimeter accuracy

V – Basic spheroidal formula

RESULTS

Accordingly, a comparison between the transformation results of geodetic coordinates obtained by using satellite observation at points of the geodetic network in Lebanon into plane rectangular in the Mercator projection (WGS-84 ellipsoid) between the modified algorithm and the traditional method of coordinate conversion are shown in Table 2.

Table 2: Results of the transformation of geodetic coordinates (B, L, ellipsoid WGS-84) into plane rectangular coordinates (x, y) in the Mercator projection

Geodetic coordinates		Traditional algorithm		Modified algorithm	
B	*L*	*x*, m	*y*, m	*x*, m	*y*, m
33° 06' 53.640"	35° 24' 58.680"	725454.800	3666623.016	725454.800	3666622.979
33° 09' 10.080"	35° 15' 35.640"	710767.932	3670500.440	710767.932	3670500.411
33° 20' 15.000"	35° 18' 12.960"	714392.733	3691072.589	714392.733	3691072.566
33° 14' 51.360"	35° 30' 20.520"	733445.984	3681536.176	733445.984	3681536.141
33° 22' 22.440"	35° 45' 14.400"	756218.787	3696016.783	756218.787	3696016.743
33° 26' 13.920"	35° 33' 12.240"	737375.735	3702672.400	737375.735	3702672.374
33° 29' 47.400"	35° 23' 25.800"	722076.805	3708888.827	722076.805	3708888.809
33° 39' 02.520"	35° 29' 00.960"	730317.636	3726194.521	730317.636	3726194.508
33° 34' 44.400"	35° 40' 47.280"	748723.365	3718696.404	748723.366	3718696.381
33° 30' 42.120"	35° 53' 09.240"	768066.424	3711745.593	768066.424	3711745.557
33° 39' 42.120"	35° 52' 57.720"	767305.582	3728376.137	767305.582	3728376.112
33° 47' 31.920"	35° 31' 59.880"	734542.254	3742000.893	734542.254	3742000.887
33° 55' 28.200"	35° 39' 29.160"	745720.228	3756967.166	745720.229	3756967.165
33° 47' 27.600"	35° 59' 03.120"	776305.504	3742986.963	776305.504	3742986.944
34° 00' 57.960"	35° 44' 35.520"	244518.560	3751508.324	244518.559	3751508.317
34° 11' 26.160"	35° 44' 21.480"	224641.158	3759646.752	224641.158	3759646.744
34° 12' 27.720"	35° 58' 40.800"	753317.260	3767335.110	753317.260	3767335.112
34° 19' 17.040"	35° 48' 03.960"	752437.731	3786682.274	752437.732	3786682.288
34° 26' 55.680"	35° 57' 29.160"	225359.641	3775758.310	225359.641	3775758.313
33° 52' 22.800"	36° 14' 16.800"	250687.907	3767439.505	250687.907	3767439.509
33° 56' 28.680"	36° 01' 14.880"	262960.887	3776957.496	262960.887	3776957.506
34° 05' 11.760"	36° 01' 24.600"	244478.734	3783879.388	244478.734	3783879.400
34° 01' 04.800"	36° 18' 00.360"	774386.503	3789196.755	774386.503	3789196.767

34° 06' 23.760"	36° 25' 49.080"	757734.904	3801347.024	757734.905	3801347.046
34° 09' 52.560"	36° 13' 41.160"	242139.383	3796787.711	242139.383	3796787.731
34° 16' 49.080"	36° 11' 56.040"	269602.325	3792519.586	269602.325	3792519.604
34° 14' 53.880"	36° 29' 53.160"	258749.488	3808410.502	258749.488	3808410.529
34° 23' 20.400"	36° 22' 33.240"	236214.689	3814083.865	236214.689	3814083.895
34° 26' 04.560"	36° 07' 45.480"	771772.909	3815890.480	771772.909	3815890.511
34° 34' 31.440"	36° 05' 25.440"	233088.840	3829806.121	233088.839	3829806.161
34° 28' 35.040"	36° 14' 29.040"	246644.787	3818434.406	246644.787	3818434.439
34° 35' 51.720"	36° 19' 47.640"	255130.738	3831672.156	255130.737	3831672.196

When comparing the computed plane rectangular coordinates by two different methods for points distributed on the Lebanese territories, it is obvious that the difference between the coordinates is within the range of 0.04 m. Thus, it is shown that the modified algorithm can be successfully used to convert geodetic coordinates obtained by GNSS technology, WGS-84 ellipsoid and Mercator projection to plane rectangular coordinates.

Currently, the proposed algorithm is developed mainly to stimulate further research in this direction for all zones of the earth's surface.

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