Time Series Analysis of Vertical Positions of High-Precision GNSS Stations in Eastern Canada: A Case Study

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SUMMARY

This study determines the rates of uplift for eleven permanent GNSS base stations in Quebec, Canada. These areas of study are close to Senneterre, Saint-Félicien and Stoneham, which are three of the regions that are well-studied for post-glacial rebound (PGR). In this study, the rate of uplift determined for Senneterre region is 6.07 mm/yr ± 0.89 mm/yr at 95% confidence level and a value of 6.60 mm/yr ± 2.42 mm/yr at 95% confidence level for Saint-Félicien area. In the areas around Quebec City, the rates vary from a maximum value of 15.54 mm/yr with 95% confidence limit of 2.92 mm/yr at the L'auvergne station to a minimum value of 5.89 mm/yr with 95% confidence limits of 2.69 mm/yr at the Chauvigny station. It should, however, be pointed out that rates of uplift determined in this study are referenced to the NAD83 Canadian Spatial Reference System (CSRS) of 1997, which is assumed stable between epochs of data collection. If this is the case, a large scale movement such as PGR in Quebec area should be detectable without the usual concern for datum instability, which are associated with traditional total station or levelling surveys. The analyses made in this study also assumes that the GNSS solutions derived from precise point positioning (PPP) mode are accurate enough to detect PGR. With this, the results obtained for the permanent GNSS base stations used in this study can be considered reasonable. On this basis, the PPP processing mode may be considered precise enough for estimating the PGR for areas with large movements like Quebec.

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1. INTRODUCTION

Post glacial rebound (PGR) was chosen as the main focus of this project because of the many significant effects it has on land surveying all over the world. The rising, subsiding, and horizontal shifting of the earth's crust due to PGR affect the positions of benchmarks, which are used for both small and large scale surveying projects. On a global scale, according to Cohen andVaníček(2013), PGR changes the inertia tensor of the earth, which affects the earth's instantaneous rotational axis.

1.1 Location of Study and Data Sources

The area of focus for this project is Quebec, around Hudson Bay, where previous studies (Vaníček and Halmiton, 1972; Koohzare, et. Al., 2008; Cohen and Vaníček, 2013) have shown that the effects of PGR in Canada are largest. Another reason for choosing Quebec as the area of focus is that the results of previously studys of the area are readily available for comparison.

This project uses precise GNSS data from 2015 to 2019 from eleven permanent GNSS base stations listed in Table 1, which also includes the municipality of the base stations, and their four-letter code. In the previous study made by Vaníček and Hamilton (1972), however, differential levelling data spanning the period of 1919 to 1964 was used; the site of their study and this study is shown in Figure 1. In the figure, the study sites (Senneterre, Stoneham and Saint-Félicien regions) by Vaníček and Hamilton are located in green.

The summary of the time series for this project is shown in Table 2, where each column consists of the year of interest with the numbers in each column representing specific GNSS days since the first day of January in the given year. A full 24 hours of RINEX data were provided for each of these days by Smartnet North America and the Government of Quebec as shown in Table 1. The authors received the RINEX files for this project via ftp server.

1 Station municipalities, codes, and GNSS RINEX file providers.

Municipality	Code	Provider
Rouyn-Noranda, QC	ROUY	Quebec Government
L'auvergne, QC	ATRI	Quebec Government
Chauvigny, QC	CHIC	Quebec Government
La Tuque, QC	LATU	Quebec Government
Amos, QC	AMOS	Smartnet
L'ange Gardien, QC	ANGE	Smartnet
Cadillac, QC	CADI	Smartnet
Alma, QC	QCAL	Smartnet
Senneterre, QC	QCSN	Smartnet
Saguenay, QC	SAGE	Smartnet
Val-Belair, QC	VALB	Smartnet

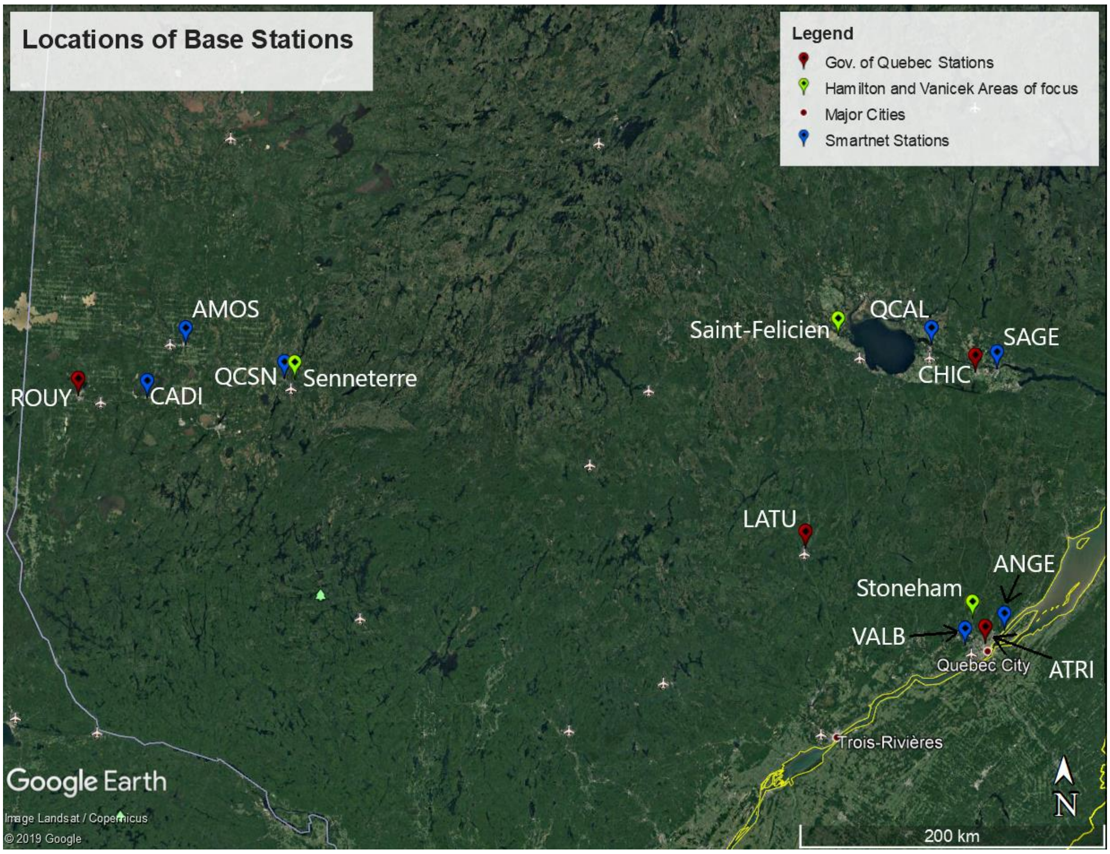

Figure 1 Locations of base stations for this project (retrieved from google earth).

Table 2 Summary of time series from the chosen GNSS base stations.

2015	2016	2017	2018	2019
60	55	49	44	36
150	145	139	134	
240	235	229	222	
330	325	319	314	

2. METHODOLOGY AND DATA PROCESSING

2.1 Data Processing Method

The GNSS raw observation data (RINEX files) were processed using a free online tool called Precise Point Positioning (PPP) service. This tool is an online application which is provided by Natural Resources Canada for post-processing GNSS data. It allows for the computation of consistent high accuracy positions from raw observation data by using precise GNSS satellite orbit ephemerides. Users can submit their RINEX observation data for either single or dual-frequency receivers that have been operated in either static or kinematic mode. Some of the PPP solutions are positions provided in Canadian Spatial Reference System (CSRS) and the International Terrestrial Reference Frame (ITRF) (Natural Resources Canada, 2019).

One important advantage of using PPP is that it utilizes a global coordinate system, which can be considered a stable datum between epochs of data collection. That way, if a large scale movement occurs, such as PGR, the movement can be detected without the usual concern for datum instability, which are associated with traditional total station or levelling surveys. In this project, the latitude, longitude, ellipsoidal height, and orthometric height of the eleven permanent GNSS base stations were determined from the PPP solutions for every 90th day. The orthometric height values , however, are considered in determining the rate of post glacial rebound at each station.

2.2 Reference Coordinate System

The North American Datum of 1983 (NAD83) CSRS 1997 was used as the coordinate system for all data processing in PPP since it is the currently accepted coordinate system in Quebec. For the vertical datum, the hybrid geoid HTv2.0 epoch 1997 was chosen since it is compatible with the Canadian Geodetic Vertical Datum of 1928 (CGVD28), which is used in Quebec.

2.3 Time Series Analysis

The orthometric height difference values derived from the PPP solutions for each of the GNSS base stations were plotted and the best-fitting regression equations as functions of time were determined from the plots. The orthometric height difference values were determined by subtracting the orthometric height value of the first day of the time series from all the orthometric height values corresponding to later days in the time series. The curve-fitting of the orthometric height difference values, including the regression analyses were done using MATLAB software package (MathWorks, 2013). The plots for the eleven base stations are given in Appendix 1, where *dH* represents the orthometric height difference value.

2.3.1 Curve Fitting and Adjustment Algorithm

As it can be seen in the Appendix 1, the graphed time series at each base station behaved like a sine wave. This is likely due to seasonal variations, since the length of each wave cycle is about one year. The effect of seasonal variations must be accommodated in fitting the curves to the plotted data. In fitting the best curve to the time series at each base station, the MATLAB interactive curve fitting tool was used. The general equation found to be best for the time series is

$$y = a_1 t + a_2 \sin(a_3 t + a_4) + a_5 t^2 \quad (1)$$

where a_1, a_2, a_3, a_4, a_5 are the unknown coefficients to be determined using different MATLAB curve fitting options and nonlinear adjustment algorithms, and t is the time of data collection in days; the effects of seasonal variations shown in the plots are accomodated in the general equation using sine function.

Equation (1) is a nonlinear model that requires nonlinear least squares adjustment methods. According to Gavin (2013), "Nonlinear least squares methods iteratively reduce the sum of the squares of the errors between the function and the measured data points through a sequence of updates to parameter values". Nonlinear models require reasonable starting values for the coefficients of the model. In this case, the initial estimates of the coefficients are chosen for the software by visually evaluating the plotted data points; the software then iteratively adjusts the coefficients using Trust-Region or Levenberg-Marquardt algorithms (MathWorks, 2013). If reasonable fit is not achieved with the initial starting points, chosen algorithm and the specified convergence criteria, different options are selected until the desired results are achieved by trial and error process.

The Levenberg-Marquardt method uses the gradient descent and Gauss-Newton minimization methods. According to Gavin (2013),

> *In the gradient descent method, the sum of the squared errors is reduced by updating the parameters in the steepest-descent direction. In the Gauss-Newton method, the sum of the squared errors is reduced by assuming the least squares function is locally quadratic, and finding the minimum of the quadratic.*

According to Ye (2014), the Trust Region method begins by setting a region around the existing best solution, where a model can somewhat approximate the original objective function. The Trust Region method then moves forward based on the model depicted within the established region. If a decrease is achieved, the approximating model is seen to represent the original function well. If a negative improvement is made, or one that is too subtle, then the model is seen to be a bad representation of the original function. The region gets smaller and smaller based on the previous improvements.

The robust-fitting options are also possible with the Levenberg-Marquardt and Trust Region algorithms. "Robust fitting is an extension of standard regression...that can even out individual outliers in a data set and neutralize their effect on the ultimate result" (idbs, 2020). The two robust methods associated with the curve-fitting toolbox in MATLAB are Least Absolute Residuals (LAR), which finds a curve that minimizes the absolute difference of the residuals, and the Bisquare method, which uses an iteratively reweighted least squares algorithm, where the weight given to each data point depends on how far the point is from the fitted curve. The main advantage of robust-fitting is that it minimizes the influence of outliers in data, which is usually a problem with ordinary least squares fitting.

3. ANALYSES OF RESULTS

In each curve-fitting process of this study, different combinations of robust-fitting options, algorithms, and equations were experimented on until a combination gave the best goodness of fit in terms of the MATLAB R-squared value and the lowest root mean squared error (RMSE). In some cases, some outliers had to be removed based on their large residual value in order to get a lower RMSE. The properties of the best-fitting regression curves for the eleven base stations are given in Table 3. As shown in this table, the best-fitting regression curves for only stations CADI, QCAL and QCSN are based on robust-fitting methods with Trust-Region algorithm and the goodness of fit for all stations is between 70% and 90%.

Table 3 Properties of best-fitting regression curves.

Station	**Goodness of Fit (R-Square)**	**RMSE (mm)**	**Adjustment Algorithm**
ATRI	0.8908	2.60	Non-robust Levenberg-Marquardt
AMOS	0.7115	3.88	Non-robust Levenberg-Marquardt
ANGE	0.7725	3.46	Non-robust Levenberg-Marquardt
CADI	0.9431	1.43	LAR Trust-Region
CHIC	0.8558	2.58	Non-robust Levenberg-Marquardt
LATU	0.8742	2.61	Non-robust Levenberg-Marquardt
QCAL	0.8388	2.40	LAR Trust-Region
QCSN	0.978	0.80	Non-robust Levenberg-Marquardt
ROUY	0.9336	1.46	Bisquare Trust-Region
SAGE	0.8872	2.20	Non-robust Levenberg-Marquardt
VALB	0.7643	3.48	Non-robust Levenberg-Marquardt

At this point, the adjusted coefficient values a_1 to a_5 in Equation (1) were determined and analyzed. The a_1 value represents the vertical velocity (or rate of uplift) of the GNSS base station, while the a_5 value represents the vertical acceleration of the GNSS base station. The summary of the rates of uplift determined for the eleven GNSS permanent stations, with

respect to NAD83 CSRS 1997, are given in Table 4. The assumption made in the adjustment algorithm is that the measurements for each station have equal relative weights.
The plots for the eleven base stations are given in Appendix 1, where *dH* (mm) represents the orthometric height difference value in millimeters and the *Residuals* (mm) are calculated as the original data values minus the corresponding values from the fitted curve.

Table 4 Computed rates of uplift and their 95% confidence limits.

Station Code	Rates of Uplift (mm/year)	95% Confidence Limits (mm/year)
ATRI	15.54	2.92
AMOS	7.98	4.02
ANGE	10.21	3.65
CADI	13.19	1.46
CHIC	5.89	2.69
LATU	10.39	2.51
QCAL	6.60	2.42
QCSN	6.07	0.89
ROUY	9.34	1.59
SAGE	7.55	2.36
VALB	12.36	4.12

The rates of uplift calculated near Senneterre and Saint-Félicien are overall very similar to the directions (upwards) and magnitudes determined by Vaníček and Hamilton (1972) as shown in Table 5. Since station QCSN is located in Senneterre, its rate is used in Table 5; in the case of Saint-Félicien, the value at the closest station (QCAL) is considered; and for Stoneham, the value at the closest station (VALB) is used. As it can be seen in Table 5, the rate around Stoneham is significantly different from the one by Vaníček and Hamilton (1972). Statistically, the results for Senneterre and Saint-Félicien regions are compatible, considering their confidence limits and the suspected deceleration on the rate of uplift estimated in this study for these regions. It should, however, be pointed out that uplift rates determined in this study are referenced to the NAD83 Canadian Spatial Reference System (CSRS) of 1997, while those by Vaníček and Hamilton are referenced to the City of Quebec. This may partially explain why the uplift rates calculated for Stoneham and other points around Quebec City are different from their values. In the areas around Quebec City, the rates vary from a maximum value of 15.54 mm/yr with 95% confidence limit of 2.92 mm/yr at the L'auvergne station to a minimum value of 5.89 mm/yr with 95% confidence limits of 2.69 mm/yr at the Chauvigny station. Considering the compatibility of the results in this study with those for the regions of Senneterre and Saint-Félicien determined by Vaníček and Hamilton and also the assumption that the NAD83 CSRS 1997 datum used in the data processing in precise point positioning (PPP) mode is global and stable between the epochs considered, the results obtained for the other stations can be considered to be equally reasonable. On this basis, the PPP processing mode may be considered precise enough for estimating the PGR for areas with large movements like Quebec.

The decceleration at each GNSS station is summarized in Table 6. The rate of PGR seems to be slowing down over time as indicated by the determined deceleration values for each station, shown in Table 6.

Table 5 Comparison of results with those of Vaníček and Hamilton (1972).

Region	**Vaníček and Hamilton**	**Current Project**
	Rates with 95% Confidence Limits (mm/year)	**Rates with 95% Confidence Limits (mm/year)**
Senneterre	8.35 ±2.28 (upwards)	6.10±0.89 (upwards)
Saint-Félicien	5.88±1.25 (upwards)	6.60±2.42 (upwards)
Stoneham	4.57±1.45 (downwards)	12.36±4.12 (upwards)

Table 6 Deceleration and 95% confidence limits of the GNSS base stations.

Station Code	**Acceleration (mm/year^2)**	**95% Confidence limits (mm/year^2)**
ATRI	-3.56	1.05
AMOS	-2.61	1.28
ANGE	-1.93	1.15
CADI	-3.34	0.49
CHIC	-1.08	0.89
LATU	-2.10	0.83
QCAL	-1.23	0.82
QCSN	-1.06	0.30
ROUY	-2.03	0.52
SAGE	-1.29	0.78
VALB	-2.44	1.29

4. CONCLUSIONS

It can be concluded that the orthometric height difference values derived from the GNSS data processed in PPP mode are due to random process and are not systematic in nature, but are due to crustal movements as previous studies had shown. In this study, seasonal variations were detected in the time series, which are obviously due to different seasons of the year in which the data was collected. This study, apart from supporting the past studies that there are large crustal movements in the region of Quebec, also suggests that the movement is still ongoing, but with possible deceleration. Since GNSS data processed in PPP mode is able to detect vertical uplifts in the Quebec region, it is recommended that the tool be further explored in studying the PGR in Quebec. One important advantage of using PPP is that it utilizes a global coordinate system, which can be considered a stable datum between epochs of data collection. In this case, large scale movement such as PGR in the Quebec area can be detected without the usual concern for datum instability, which is associated with traditional total station or levelling surveys.

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REFERENCES

Cohen, S. C. and Vanicek, P (eds.) (2013). Slow Deformation and Transmission of Stress in the Earth. ISBN: 978-1-118-66630-2, March. *American Geophysical Union*. 138p.

Gavin, H. P. (2013). *The Levenberg-Marquardt algorithm for nonlinear least squares curve-fitting problems.* Taken from the Department of Civil and Environmental Engineering. October 9. http://people.duke.edu/~hpgavin/ce281/lm.pdf

idbs. (2020). *Robust Fitting and Complex Models*. Taken from idbs: https://www.idbs.com/excelcurvefitting/best-practice/best-practice/robust-fitting-and-complex-models/

Koohzare, A; Vanicek, P; Santos, M. (2008). *Pattern of Recent Vertical Crustal Movements in Canada*. Taken from the Journal of Geodynamics. Fredericton, New Brunswick. February 19.

MathWorks (2013). *Curve Fitting ToolboxTM User's Guide*. R2013a. MATLAB. The MathWorks, Inc. March.

Natural Resources Canada. (2019). *Tools and Applications*. February 22. https://www.nrcan.gc.ca/maps-tools-and-publications/tools/geodetic-reference-systems-tools/tools-applications/10925#ppp

Vaníček, P. and Hamilton, A. (1972). *Further Analysis of Vertical Crustal Movement Observations in the Lac St. Jean Area, Quebec.* Taken from the Canadian Journal of Earth Sciences. Fredericton: University of New Brunswick.

Ye, W. (2014). *Trust-region methods*. Hentet fra Northwestern: https://optimization.mccormick.northwestern.edu/index.php/Trust-region_methods

BIOGRAPHICAL NOTES

Wesley Brown is a Bachelor of Science student at the British Columbia Institute of Technology. He is in his fourth year of the Geomatics Degree program. Wesley has three consecutive summers of work experience in the surveying field. One summer as a survey assistant, and two summers as a crew chief doing construction and legal surveys. After graduating from BCIT in April, he plans on articling right away with the goal of becoming a British Columbia Land Surveyor.

John Ogundare is a practising professional geomatics engineer in British Columbia, Canada; an instructor of Geomatics engineering at the British Columbia Institute of Technology, Canada; and an author of two books published by Wiley & Sons, Inc., Hoboken: *The Precision Surveying: The Principles and Geomatics Practice* and *Understanding Least Squares Estimation and Geomatics Data Analysis*. He has been in the field of geomatics for over thirty years in Africa and Canada. Dr. Ogundare has been a representative of and a special examiner for the Canadian Board of Examiners for Professional Surveyors (CBEPS) for over ten years.

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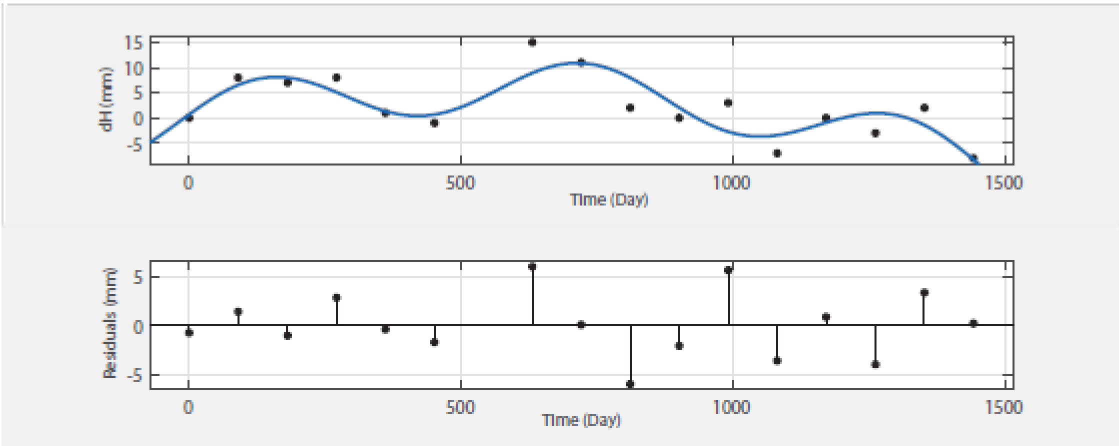
Appendix

Plots of orthometric height difference values dH (mm) against time (Day), and of the *Residuals (mm)* calculated as the original data values minus the corresponding values from the fitted curve against *time (Day)*, for the eleven GNSS base stations used in this study.



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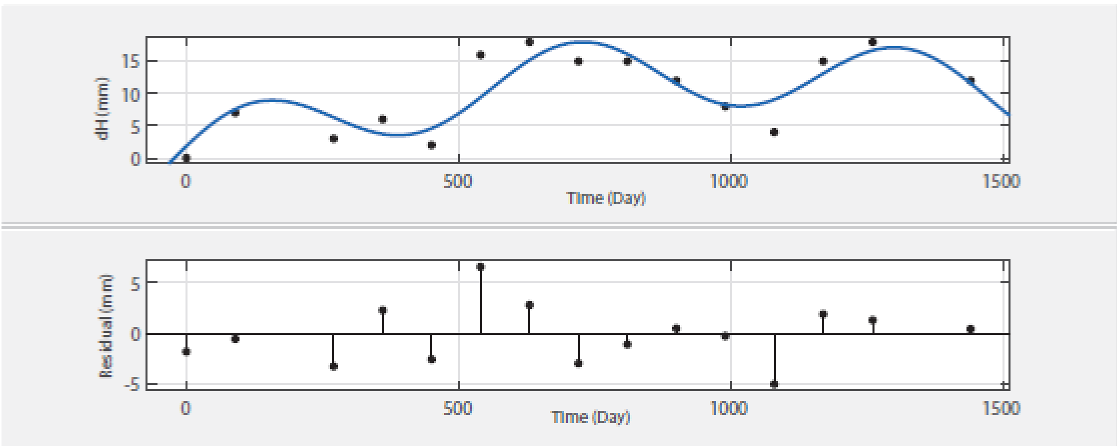

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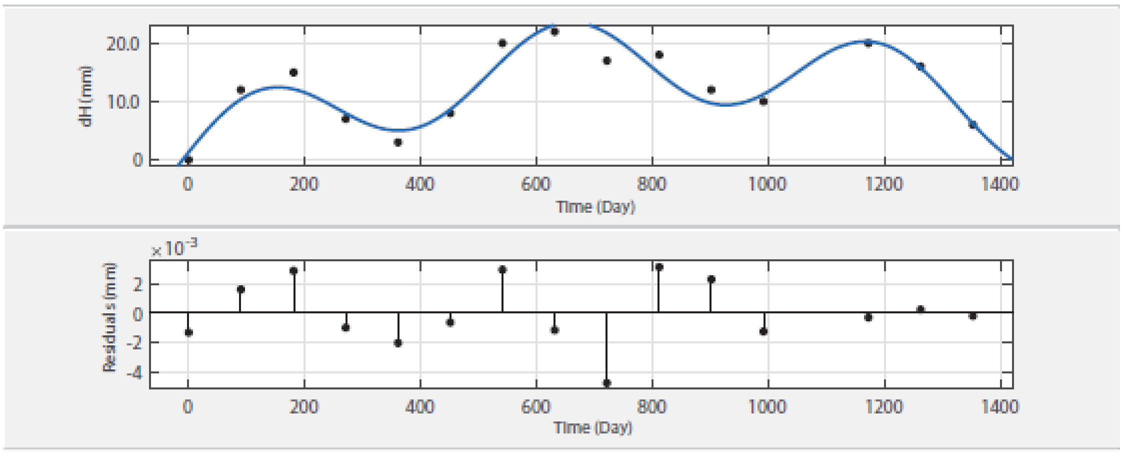

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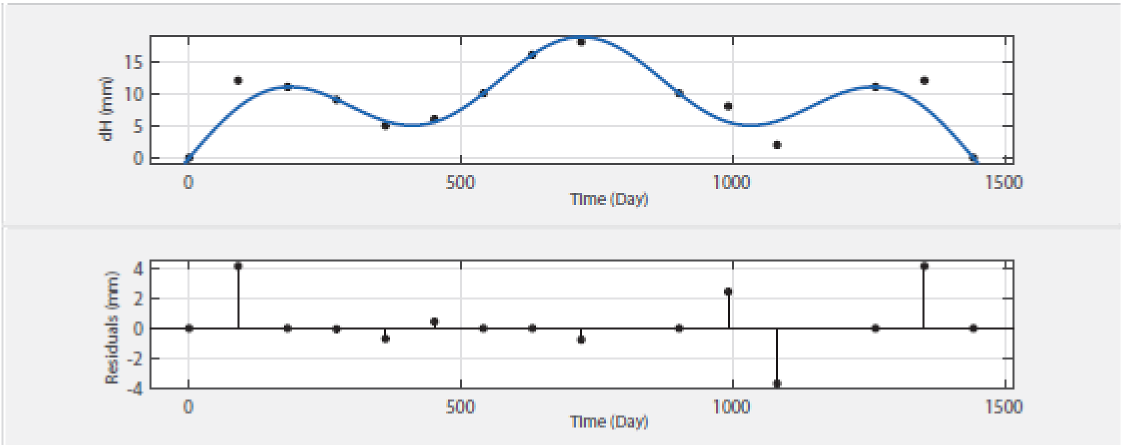

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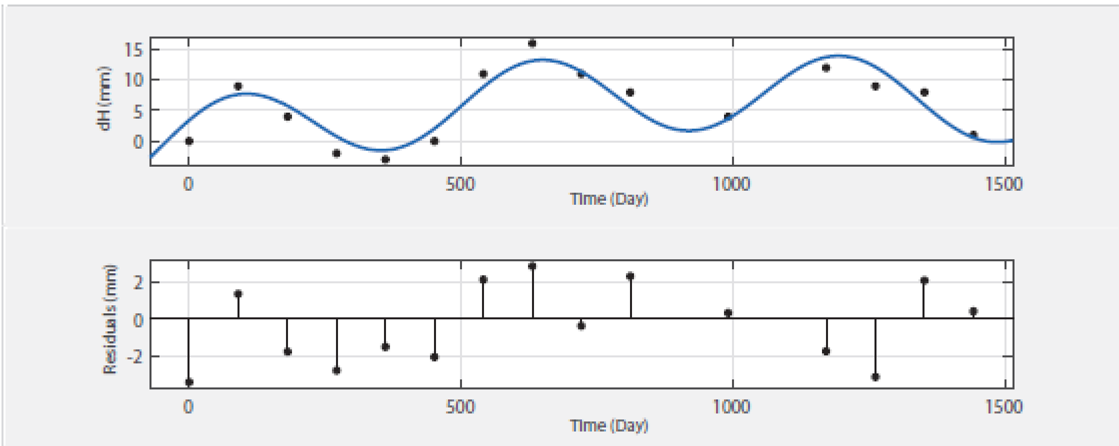

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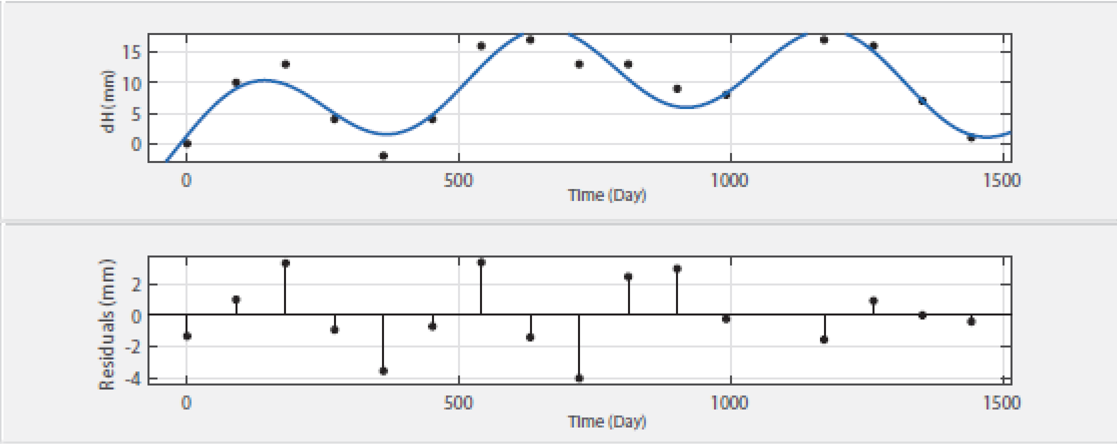

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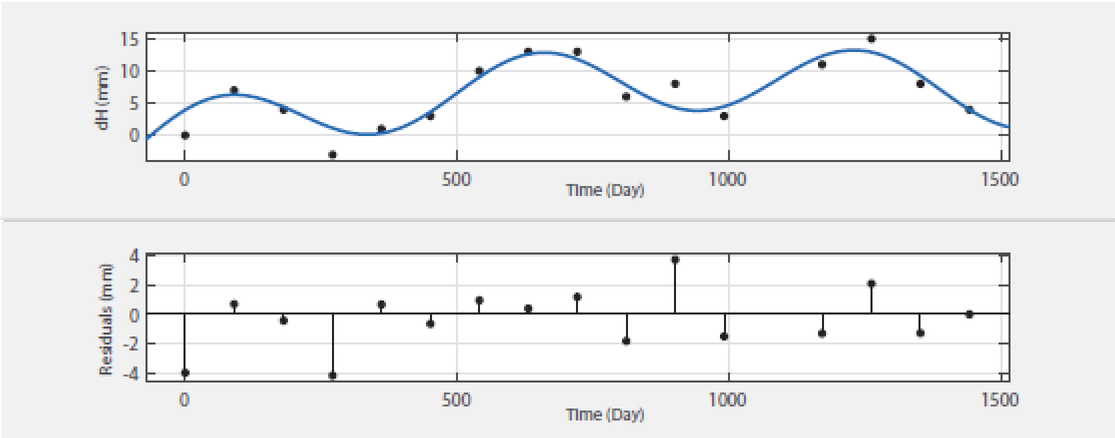

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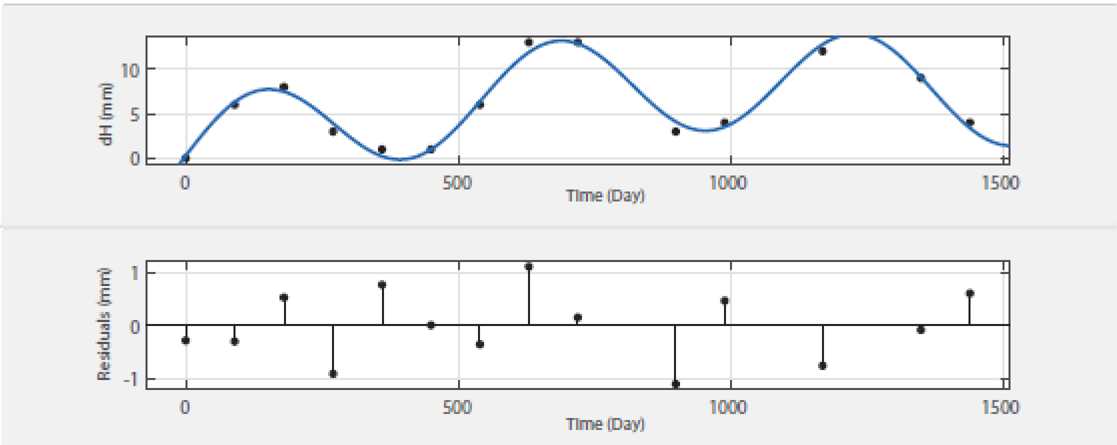

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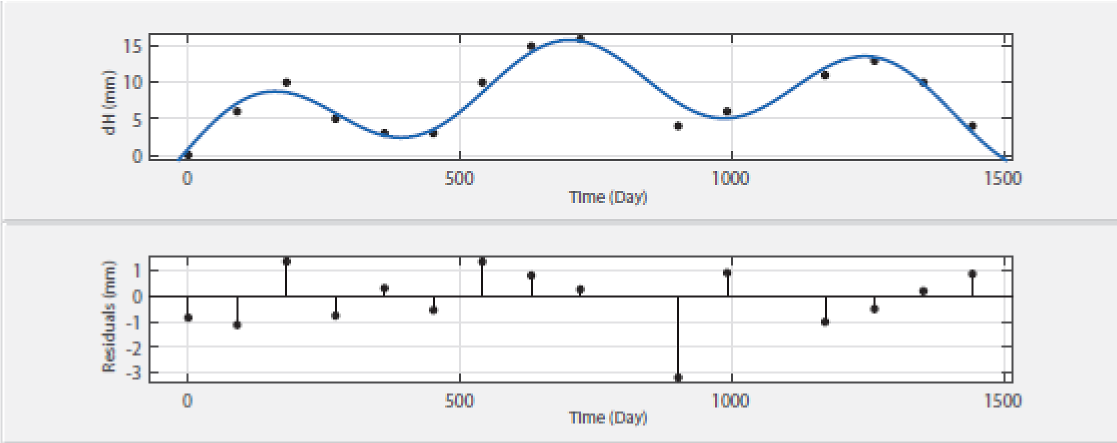
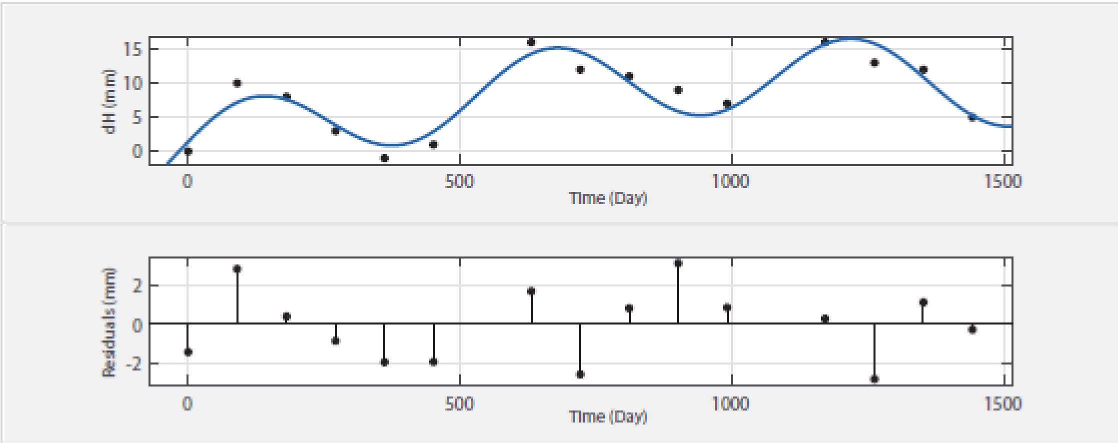


Rouyn-Noranda



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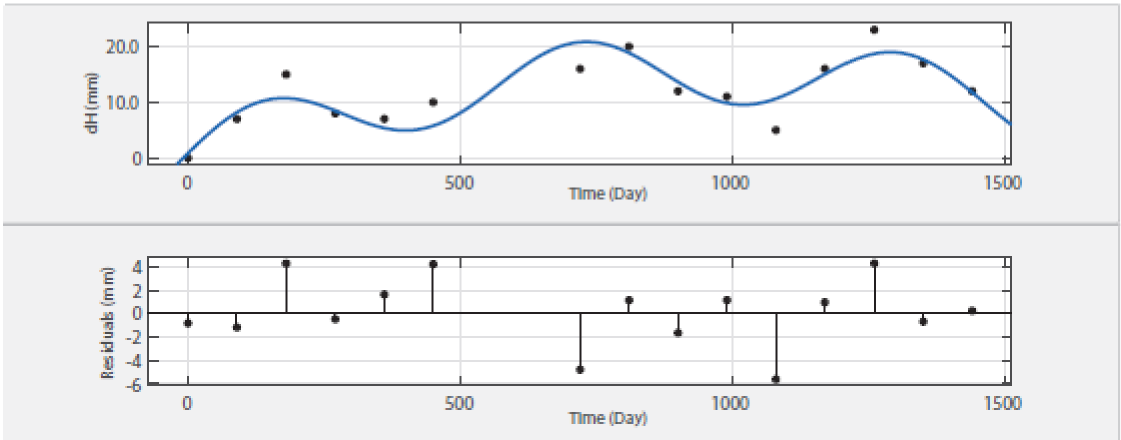

Val-Belair