

Coordinates Trajectory Models: Basics and Implementation

Mohammed ALJAFAR, Saudi Arabia

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SUMMARY

This technical paper discusses the use of trajectory models in GNSS station coordinates' time-series. The author starts by introducing initial trajectory models and their evolution to the modern state. A case study using three years of 16 Ohio GNSS stations data is provided. In the study, the author explains how the daily raw GNSS data is processed to get the daily solutions. Then, the author applies to the processed data the constant velocity model (CVM) with and without jumps, and the standard linear trajectory model (SLTM) with and without outliers. A demonstration of how the SLTM model uses the weighted least squares adjustment to estimate the model's parameters. The author describes the outlier detection method implemented in the study. After that, a comparison of each model residuals is provided as well as discussion of results.

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1. INTRODUCTION

When we think of the coordinates of a point in a 3D space, we usually think of these coordinates as a means of describing the position of this point somewhere near three axes. These axes are typically perpendicular and connected to each other at the origin of the space center where the coordinates of the center become (0, 0, 0). The common symbols of these axes are X, Y, and Z and the coordinates values of any point represent the distances from the origin of the axes to the position of the point projected on each axis. This system is usually referred to as a cartesian coordinate system and used to describe the location of any object in that 3D space. In the geospatial community, cartesian coordinate systems are commonly used to describe locations of objects, such as buildings or roads, on the surface of the Earth. When a coordinate system's origin is positioned and fixed at the center of the earth, is referred to as Earth Center Earth Fixed (ECEF) coordinate system.

In the past, geodesists expressed coordinates of positions as fixed values. Once the position was calculated and assigned, the coordinates were treated as constants through time. However, after the theory of tectonic plates became popular, the idea of constant coordinates came to be irrelevant for accurate geodetic applications (Bevis and Brown, 2014). Whereby, plate tectonic theory states that the outer layer of the Earth consists of solid plates that float in constant motion over a viscous layer (Schubert, 2015). The key point of the theory is the motion of the outer layer, which makes the concept of fixed and constant coordinates obsolete from the geodetic perspective. Of course, with the advancement of positioning technology, the tectonic plate movement became widely recognized and precisely calculated. For instance, space positioning technologies have allowed scientists to collect large amounts of precise data continuously. Of which, the Global Navigation Satellite System (GNSS) is one of the well-known technologies, where data collection is continuous around the clock. Nowadays, thousands of GNSS stations spread across the globe providing precise positioning to users in many fields. These stations are referred to as Continuously Operating Reference Stations (CORS).

The concept of motion is always coupled with velocity, where velocity measures the rate of motion. When we are talking about velocity, which is the change of position over time, we introduce the time dimension to our coordinate system including coordinates that are usually collected from CORS stations. It is difficult for humans to imagine any space that is larger than three dimensions. However, by taking a simple approach, visualizing the coordinates in four dimensions becomes relatively easy. This approach is simply taking each dimension of the 3D (X, Y, or Z) and projecting it on the time dimension. By doing so, we obtain the so-called trajectory model (Bevis and Brown, 2014).

Multiple trajectory models have evolved throughout histories such as the Standard Linear Trajectory Model (SLTM) and Extended Linear Trajectory Model (ELTM). These models are usually assigned to fixed CORS stations where data is collected and processed routinely. The

goal of applying these models to the CORS stations is to understand the motion of the station, and eventually the location where the station resides, over an extended period. Therefore, this paper sheds the light on the advancement and the use of the trajectory models from the geodetic perspective including the SLTM and ELTM models. Besides, a demonstration of applying the SLTM model as well as older models on three years of continuous daily GNSS observations of 16 CORS stations located across the state of Ohio is provided.

2. THE FUNDAMENTALS OF TRAJECTORY MODELS

The idea of realizing a reference system, that is fixing coordinates of reference system at a specific moment in time to active or passive geodetic control points, is called a reference frame. Historically, reference frames were thought of as frames that have constant values. However, with the popularity of the crustal motion theory of the Earth, the velocity of reference stations must be calculated along with the fixed position at a moment in time to improve the accuracy of fitting the data to a model. Geodesists and geophysicists have a major interest in modeling the movement of a station. While geophysicists are interested in studying the dynamics of the crustal motion at the CORS station, geodesists are much more keen on predicting the position of the station and the accuracy of the past and predicted position (Bevis and Brown, 2014).

Figure 1 shows an early model that represents the position of a station and velocity, which is referred to as the Constant Velocity Model (CVM). In the figure, the CVM model is applied to daily position solutions for COLB station, which is part of the Ohio Department of Transportation (ODOT) CORS network. The model consists of two parts with each part consisting of three subparts. The main parts are a fixed position and

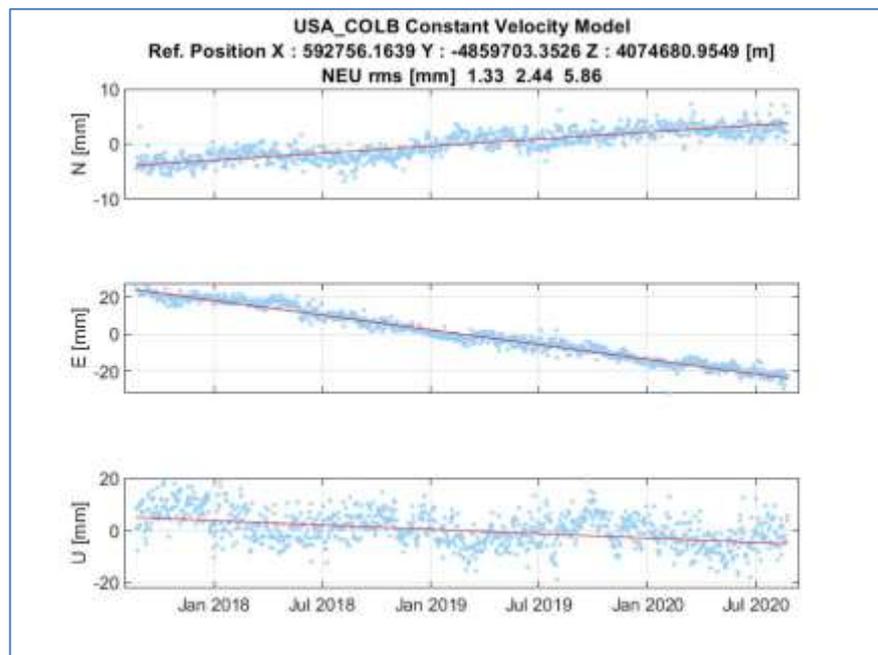


Figure 1: Colb Station Constant Velocity Model

velocity, and the subparts are the X, Y, and Z. Or for better illustration topocentric North, East, and Up. The model estimates the best fit line using the least-squares adjustment method. In the figure, the light blue dots represent the scatter of the daily solution, and the red lines

reflect the model's best fit line. Bevis and Brown (2014) illustrate the model as a function of time given as:

$$X(t) = X_R + V_x(t - t_R)$$

$$Y(t) = Y_R + V_y(t - t_R) \quad (1)$$

$$Z(t) = Z_R + V_z(t - t_R)$$

where:

$X(t)$, $Y(t)$, and $Z(t)$ are the estimated position coordinates at time t

X_R , Y_R , and Z_R are the reference position coordinates

V_x , V_y , and V_z are the velocities associated with each component

t is the time of observation

t_R is the reference time

Model (1) can be represented as a vector equation:

$$x(t) = x_R + v(t - t_R) \quad (2)$$

where:

$$x(t) = \begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix}$$

$$x_R = \begin{bmatrix} X_R \\ Y_R \\ Z_R \end{bmatrix}$$

$$v = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}$$

In the geodetic field, the reference time is defined differently. For example, the International Earth Rotation and Reference Systems Service (IERS) sets one reference time for all stations. However, other entities such as OSU dedicate each station with its own reference time, which is usually set to the mean of the observation time series (Bevis and Brown, 2014). Even though this model captures an approximation of the station motion using a line fitting approach, Earth's motion is much more complicated to be represented in this model.

One of the aspects this model lacks is the inability to accommodate any type of instant variations in the model. These variations can come in the form of jumps whether artificial or natural. An artificial jump is usually defined as any change in the CORS station position that is not caused by the change of the location where the station resides. For example, a change or replacement of the station antenna is an artificial jump. If not properly accounted for, this jump can negatively affect the values of the trajectory model. This can happen by giving the illusion of change in the station's position, where the station does not move. The false change will be included in the calculation of the velocity and reference position, leading to an inaccurate model. Natural jumps are the displacement of the station's position due to a natural phenomenon such as earthquakes and volcanoes (Bevis and Brown, 2014).

To overcome the instant jump limitation of the CVM, another term called the Heaviside function is added to the model. The user can specify the time of the jump and this function

would estimate the magnitude of the jump during the least square adjustment. The Heaviside function is defined as (Bevis and Brown, 2014):

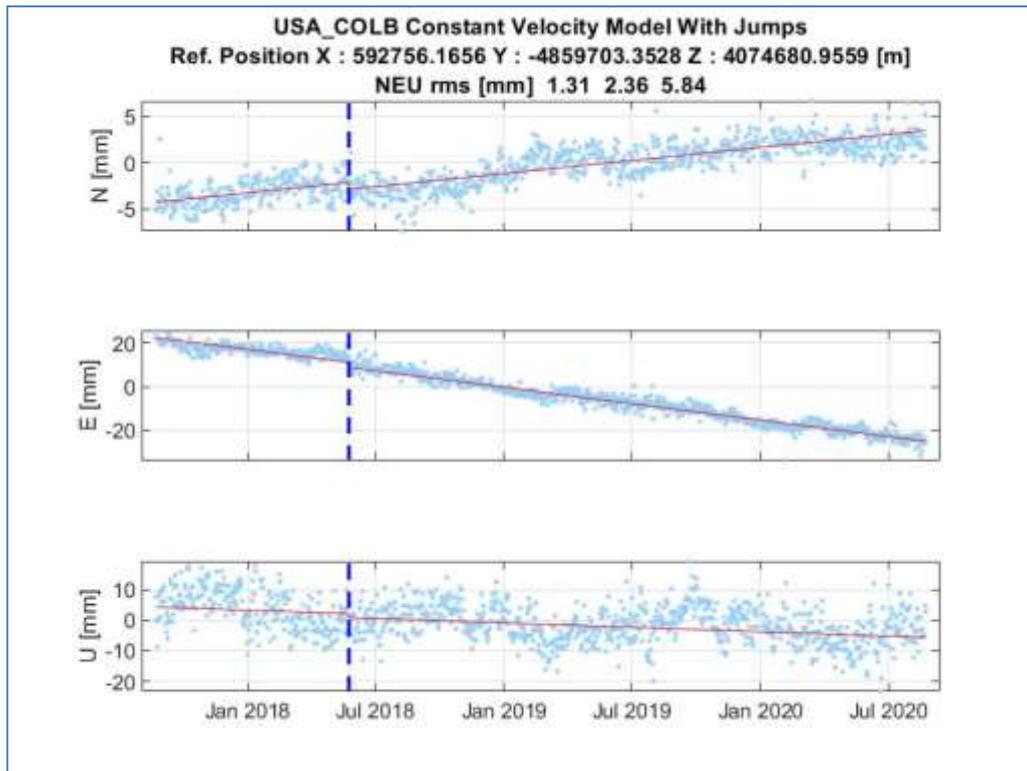


Figure 2: COLB Station Constant Velocity Model With Jumps

$$\begin{aligned} H(t) &= 0 && \text{for } t < 0 \\ H(t) &= 1/2 && \text{for } t = 0 \\ H(t) &= 1 && \text{for } t > 0 \end{aligned} \quad (3)$$

The modified CVM is expressed as:

$$x(t) = x_R + v(t - t_R) + \sum_{j=1}^{n_j} b_j H(t - t_j) \quad (4)$$

where

n_j is the number of jumps

t_j is the time of the jump

b_j is the jump magnitude to be estimated

Figure 2 illustrates the modified CVM model to accommodate jumps. The vertical blue dashed line indicates the time of an artificial jump in the COLB station. To illustrate the modeled instant shift in position, the red line in the figure breaks in June 2018 where the blue dashed line is apparent.

Standard Linear Trajectory Model:

Even with the modified model, some stations' crustal motions were not modeled properly due to the simplistic nature of both the regular and modified CVM models. The SLTM extends the predecessor models by adding two additional segments, which increases the versatility of the

model. The first segment is a truncated four-term Fourier series that models annual and semiannual oscillation. These oscillations are due to the elastic reaction of the Earth to change in cyclical changes of atmosphere and load distribution (Bevis and Brown, 2014). The addition of the four-term Fourier series to the model is illustrated as:

$$x(t) = x_R + v(t - t_R) + \sum_{j=1}^{n_j} b_j H(t - t_j) + \sum_{k=1}^{n_f} [s_k \sin(\omega_k t) + c_k \cos(\omega_k t)] \quad (5)$$

where

n_f is the number of frequencies

$$\omega_k = 2\pi/\tau_k$$

τ_k is the periodic cycle e.g. 1 year, 1/2 year, etc.

The last component to be added to generate the SLTM model is the polynomial trend. Up to this point, we treated the velocity as constant, which is usually not the case. According to Bevis and Brown (2014), multiple CORS stations have shown accelerating trends in the velocity of the model. With the added accelerating or trend component the SLTM model is stated as:

$$x(t) = \sum_{i=1}^{n_p+1} p_i (t - t_R)^{i-1} + \sum_{j=1}^{n_j} b_j H(t - t_j) + \sum_{k=1}^{n_f} [s_k \sin(\omega_k t) + c_k \cos(\omega_k t)] \quad (6)$$

Where

n_p is the power of the polynomial

If n_p becomes 1, then the model would have a reference position and velocity. With larger values, the model would represent the data as a regular polynomial function. It is important to find the right polynomial order in representing the model trend. The higher orders might fit the data better, but the model might be modeling noise rather than actual movement. Bevis and Brown (2014) suggest taking a more cautious approach when it comes to higher-order polynomials. In fact, they claim that most CORS stations can be modeled with n_p equals 1.

Extended Linear Trajectory Model:

Even though the SLTM model is sufficient in modeling many CORS stations, it lacks the ability to accurately model post-seismic transient deformation. After the co-seismic jump associated with an Earthquake, a diminishing movement is usually recorded lasting to several years and called transient displacement. This displacement can be modeled by a logarithmic transient formula (Bevis and Brown, 2014):

$$d = A \log(1 + \Delta t/T) \quad (9)$$

where

Δt is the period after the earthquake effect

T is a parameter considered in this model as a given constant and is estimated to be 1 year in case of one transient and (1 & 0.0523) for double transient model (Bevis et al, 2019).

The model is stated as:

$$x(t) = \sum_{i=1}^{n_p+1} p_i (t - t_R)^{i-1} + \sum_{j=1}^{n_j} b_j H(t - t_j) + \sum_{k=1}^{n_f} [s_k \sin(\omega_k t) + c_k \cos(\omega_k t)] + \sum_{i=1}^{n_T} a_i \log(1 + \Delta t_i/T_i) \quad (10)$$

Models similar to the STML that go beyond the constant velocity model in estimating GNSS timeseries positions are popular in the geodetic field. For example, The Nevada Geodetic Laboratory uses a model that incorporates the annual/semiannual oscillations and jumps in estimating the model of the timeseries positions (Blewitt et al., 2018). The same elements are part of the STML.

3. PROJECT IMPLEMENTATION

3.1 Site selection and GNSS data processing

The author's selection of the Ohio state area for implementing the project is based on two reasons. The first one is the availability of continuous GNSS data for Ohio state, and the second reason is the similarity of Ohio's seismic nature to the author's home country Saudi Arabia. Out of 39 Ohio stations available in the OSU archive, 16 stations scattered across the state are selected. Daily raw Receiver Independent Exchange Format (RINEX) files as well as processed daily solutions are retrieved from the OSU database. Three years of daily coordinates are processed spanning from day of year (DOY) 235 of 2017 to DOY 235 of 2020. Precise Point Positioning (PPP) processing of the data is done using the GPSPACE software. This software requires precise ephemeris, clock correction, and ocean loadings to be downloaded from NASA's archive of space geodesy data to precisely calculate the daily positions. Once the data is processed, daily solutions are extracted along with standard errors to be implemented in the trajectory modeling phase.

3.2 Trajectory Modeling

Once processing of the daily solutions is completed, the trajectory modeling of the data takes place. The suitable model for the Ohio area is the SLTM, since there are no major jumps with transients in this area. In fact, all the jumps in the 16 stations are artificial, i.e. due to changes in the antenna, radom, or receiver. The model formula is repeated below:

$$x(t) = \sum_{i=1}^{np+1} p_i(t - t_R)^{i-1} + \sum_{j=1}^{nj} b_j H(t - t_j) + \sum_{k=1}^{nf} [s_k \sin(\omega_k t) + c_k \cos(\omega_k t)] \quad (11)$$

After separating each coordinate's components and its sigma i.e. (x, σ_x), (y, σ_y), and (z, σ_z), each of the three components goes through two stages of weighted least squares adjustment to estimate the proper model. The weighted least squares adjustment is a parameter estimation technique that aims to minimize the sum of the squared residuals and usually expressed as (Schaffrin & Snow, 2019):

$$\hat{x} = B^{-1}(A^T C_b^{-1} b), B = A^T C_b^{-1} A \quad (12)$$

$$C_x = B^{-1} \quad (13)$$

where

\hat{x} is the vector of parameters that needs to be estimated

b is the vector of daily solution

A is the design matrix of the SLTM model

C_b is the covariance matrix for the provided observations

C_x is the covariance matrix for the estimated residuals

The design matrix A is constructed using the MATLAB function `design_sltm`. As mentioned, the reference time is the mean time of observation. However, the reference position is estimated in the first run of least squares. Once the reference position is identified, it is subtracted from all the observations to get the difference in each component. Then, for the

sake of better visualizations, the data is rotated to a topocentric coordinate system (e, n, and u) where the center is the reference position. The formula for the linear transformation is

$$R = \begin{bmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\sin\phi\cos\lambda & -\sin\phi\sin\lambda & \cos\phi \\ \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi \end{bmatrix}$$

$$v_e = Rv_x$$

where

R is the rotation matrix

v_e is the vector of rotated topocentric components

v_x is the vector of differences in the geocentric form

In addition, to propagate the residuals of the geocentric components, we can use the below expression:

$$C_e = RC_xR^T$$

where

C_e is the covariance of the topocentric data

Once the data is in the topocentric form, another least-squares adjustment is performed to calculate the proper parameters of the models. Then, the root mean squares error (RMSE) is calculated for the difference between the model and the actual data. Towards the end, the data is plotted along with the estimated model. To overcome modeling the data with the original time series that has many gaps, a uniform artificial time-series is created. This time-series take the first and last epochs of the original time series and divide the period into small uniform sections e.g. 1000 sections. By modeling the uniform time series, the presentation and shape of the model will appear in an organized and clear way. Below is an example showing the COLB station with reference position, RMSE, number of jumps, etc. shown in the title.

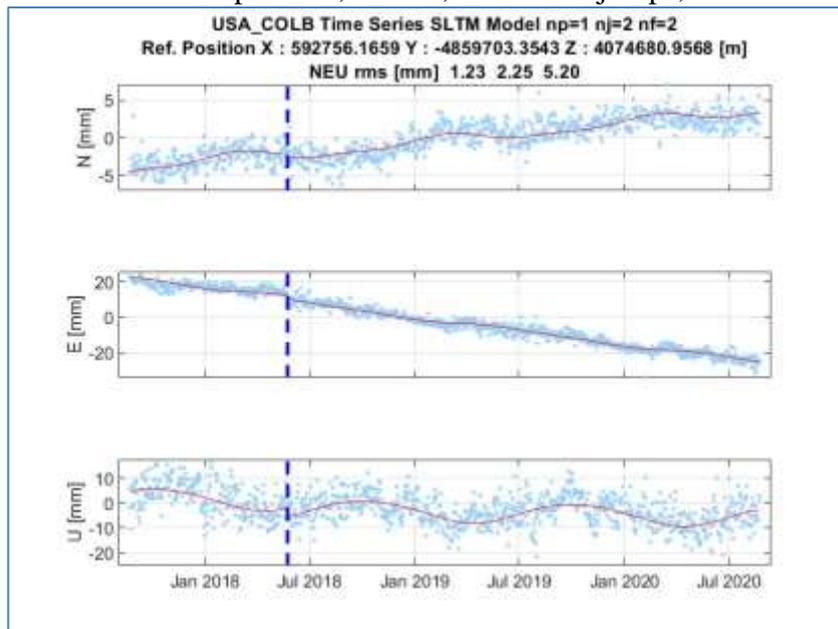


Figure 3: Colb Station Sltm Model

3.3 Outlier Detection

It is common to find outliers in GNSS data, but the outliers should be very minimal. Hence, all the data filtered for any measurement that has an error of more than 3x the RMSE for each station in this study. After the filter has been applied, the data is reproduced using the cleaned data. This way we ensure that we have clean and consistent data. As illustrated in the below plots, outliers are plotted using orange color.

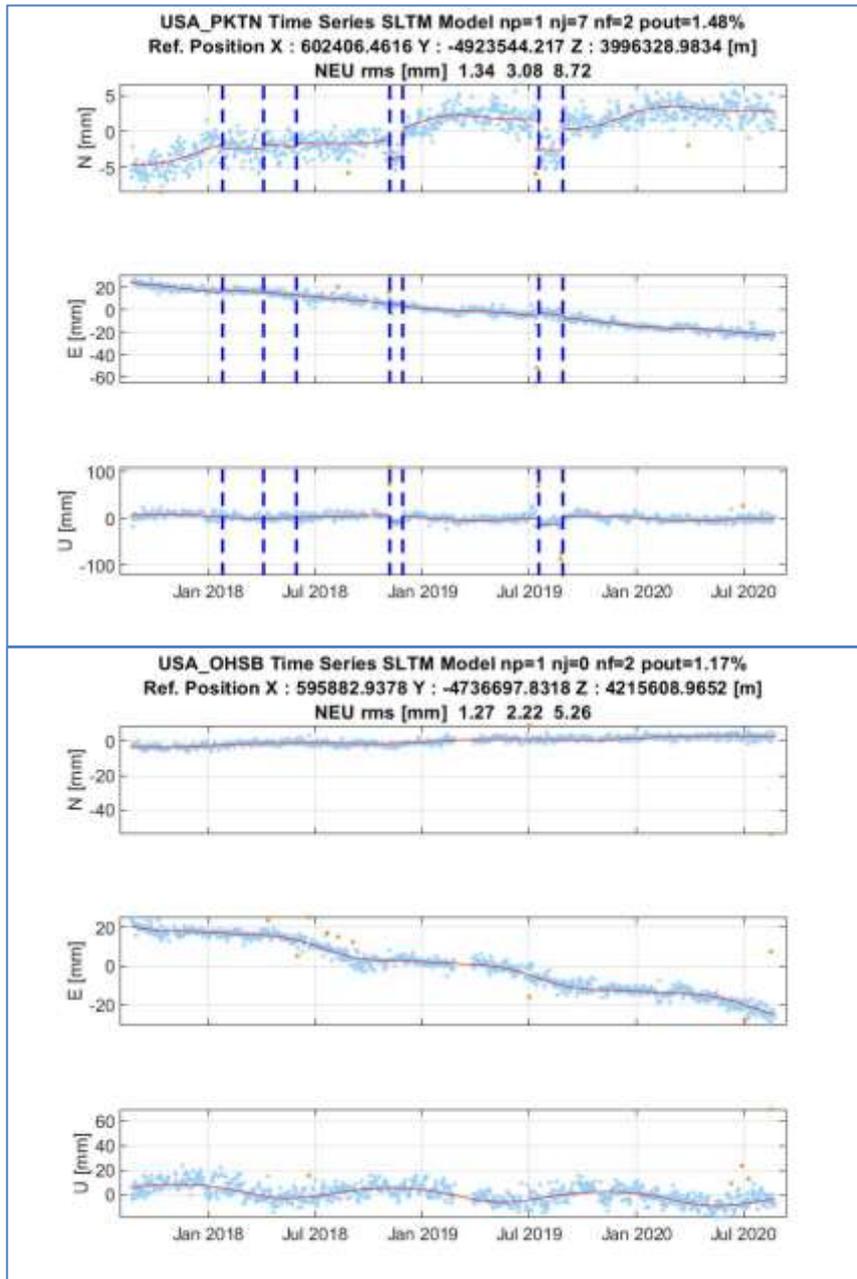


Figure 4: SLTM Without Outliers

Now that the models are created, we can use these models to better understand how the motion of each station behaves. We can answer questions such as whether there is an acceleration in the motion of some stations, or the model is just represented as a constant movement with seasonal motion. Creating such models would lead to a more accurate prediction of anticipating or approximating where the station is moving to in the future. In addition, these models help in defining more stable reference frames, as the position of the station is better understood.

3.4 Results

In this study, multiple models are implemented in the processing of the data. The CVM, CVM with jumps, and SLTM are applied to the 16 stations. It is evident that the more recent models should fit the data better. As shown in Table 1, the RMSEs of the stations get smaller with more advanced models. The CVM RMSEs are larger than the CVM's with jumps RMSEs and the same applies to the SLTM. The improvements are due to the introduction of jumps then seasonal variations. If there was any acceleration, the later models would detect the change. However, in our stations, the velocities are considered constant.

	CVM RMS in mm			CVM with Jumps RMS in mm			SLTM in mm			SLTM without outliers in mm		
	E	N	U	E	N	U	E	N	U	E	N	U
USA_COLB	2.437	1.327	5.862	2.36	1.31	5.84	2.25	1.23	5.20	2.25	1.23	5.20
USA_FREO	2.479	1.56	5.934	2.42	1.40	5.93	2.23	1.28	5.39	2.22	1.27	5.37
USA_GALP	2.362	1.275	6.092	2.31	1.26	6.06	2.12	1.22	5.54	2.12	1.22	5.54
USA_GARF	2.581	1.593	6.496	2.31	1.58	5.96	2.18	1.45	5.51	2.18	1.45	5.51
USA_LEBA	2.6	1.471	5.912	2.54	1.42	5.90	2.46	1.26	5.30	2.46	1.26	5.30
USA_MTVR	2.711	1.359	6.218	2.63	1.34	5.77	2.46	1.26	5.27	2.46	1.26	5.27
USA_OHAS	2.423	1.439	7.36	2.42	1.44	7.36	2.23	1.40	6.63	2.22	1.36	6.28
USA_OHCL	2.654	2.839	6.704	2.48	2.21	5.87	2.34	2.11	5.37	2.34	2.11	5.37
USA_OHHI	2.727	1.785	5.847	2.57	1.73	5.83	2.39	1.48	5.37	2.39	1.48	5.37
USA_OHMN	2.464	1.486	6.183	2.46	1.49	6.18	2.17	1.26	5.57	2.17	1.26	5.57
USA_OHMR	2.478	1.344	5.884	2.48	1.34	5.88	2.17	1.25	5.18	2.17	1.25	5.18
USA_OHSB	3.019	2.235	6.7	3.02	2.24	6.70	2.43	2.18	5.74	2.22	1.27	5.26
USA_OHWI	5.896	2.056	6.919	5.89	2.01	6.81	3.22	1.54	5.23	3.20	1.52	5.08
USA_PKT N	3.549	1.83	10.053	3.42	1.47	9.60	3.39	1.34	9.21	3.08	1.34	8.72
USA_STKR	5.218	3.384	10.91	4.47	2.72	10.52	4.24	2.63	9.86	3.28	2.34	8.08
USA_TIFF	2.357	1.436	5.81	2.36	1.44	5.81	1.98	1.37	5.24	1.98	1.37	5.24

Table 1: RMSE of different models

In addition, Table 2 shows the estimated magnitude of velocities at each station as well as the norm of the velocity vector. We can notice that the velocities are quite similar since all the stations are located close to each other within the North American Tectonic plate.

	Velocities mm/y			Euclidean Length
	E	N	U	
USA_COLB	-15.228	2.672	-1.328	15.518
USA_FREO	-14.616	3.300	-0.081	14.984
USA_GALP	-14.439	3.149	1.023	14.813
USA_GARF	-14.987	3.213	-3.404	15.701
USA_LEBA	-15.084	2.037	-0.929	15.249
USA_MTVR	-15.292	2.352	-1.178	15.517
USA_OHAS	-15.728	3.216	-3.485	16.428
USA_OHCL	-14.919	2.121	-0.738	15.087
USA_OHHI	-14.475	1.999	-1.429	14.682
USA_OHMN	-15.133	2.944	-2.004	15.546
USA_OHMR	-15.697	1.797	-2.494	15.995
USA_OHSB	-15.226	2.144	-3.047	15.675
USA_OHWI	-16.501	1.028	-1.789	16.630
USA_PKTN	-15.542	2.457	-3.106	16.038
USA_STKR	-14.627	2.255	0.257	14.802
USA_TIFF	-14.689	2.010	-2.286	15.001

Table 2: Estimated stations velocities.

Out of the 16 stations, only one station has an outlier percentage that is above the 3% mark, which is OHCL. As shown below, the outlier detection algorithm shows OHCL station having 3.2% outliers in the data. We can see that the north component has many red dots at the beginning of the data.

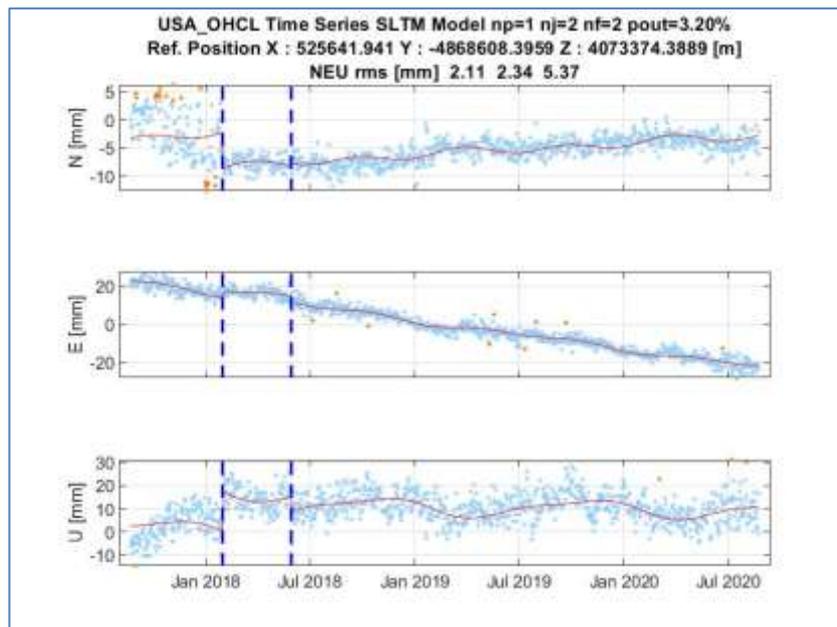


Figure 5: OHCL SLTM Plot

The reasoning of this station behavior was not identified. However, when compared to the data from Nevada Geodetic Laboratory (NGL), we can see that the NGL plot reflects this study's data rhythm. Therefore, we can say that the behavior of this station is not physical, and a thorough examination of the station is needed to identify the underlying reasoning. (Blewitt et al., 2018).

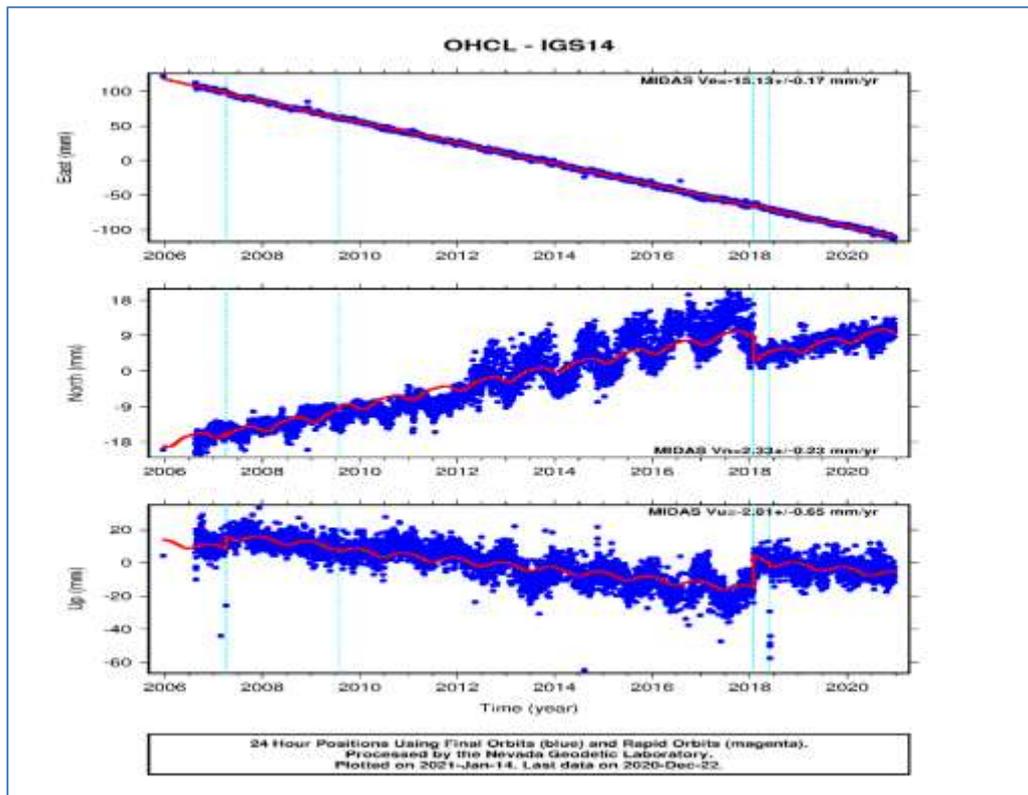


Figure 6: Nevada Geodetic Laboratory Plot of OHCL

4. CONCLUSION

In this paper, several iterations of trajectory models have been discussed. Namely, the CVM without and with jumps, SLTM, and ELTM are addressed in how they can model time-series of GNSS station observations differently. A practical implementation of the three models including the SLTM is provided. Three years of observations of sixteen Ohio CORS stations are utilized to demonstrate the effectiveness of modeling the GNSS time series. Multiple geodetic techniques are performed in this project, for example, employing PPP processing on RINEX data as well as the application of least squares adjustment on the time series.

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BIOGRAPHICAL NOTES

Mohammed Aljafar is a geomatics engineer working in the oil and gas industry in Saudi Arabia. He received a bachelor's degree in Surveying Engineering from the University of Maine and a master's degree in the Geodetic Science from the Ohio State University, USA. He is interested in the various disciplines of surveying including data collection using contemporary and traditional methods as well as data processing. His preferred topics are GNSS technology, and coordinate transformations.

CONTACTS

Mohammed Aljafar
The Ohio State University
34264, Ash Shulah Dist.
Dammam
Saudi Arabia
Tel. +966583387163
Email: aljafar.1@osu.edu and Mohammed.aljafar@outlook.com

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