Accuracy Management in GPS Engineering Applications

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ABSTRACT

GPS technology plays an essential role in contemporary engineering surveys. However, the problems of accuracy planning and evaluation are not yet fully solved concerning the precise GPS applications (deformation measurements, long term monitoring of structures). On the other hand, many practical commercial user wants to know simple ways to improve his GPS results.

This contribution presents the more than ten years experience of the authors in this field based on various experimental GPS campaigns and real project surveys. Accuracy factors of static and kinematic GPS methods are described. Problems in confrontation of formal and real accuracy are discussed. The best reduced observation scheme which can substitute the long sessions with minimal loss of accuracy is proposed together with practical results. Accuracy parameters for static surveys in respect of baseline length and observation time are evaluated. Contributions of the permanent GPS stations data and other IGS products are considered.

The accuracy management principles were successfully applied in practice, e.g. in long term geodynamic monitoring of an mountain area, in bridge and railway track deformation surveys. Another example is the improvement of vertical accuracy in GPS re-measurement of a levelling line section for the purposes of quasigeoid investigations and modeling.

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1. INTRODUCTION

Error analyses and accuracy planning play an important role in engineering surveys. For a site surveyor it is important to know about real accuracy limits of surveying method employed, so as to choose technically and economically optimal measuring technology. Accuracy management is very important for reliable and economic achievement of the surveys goals - the fulfilling of project accuracy demands.

Main current GPS applications in engineering surveys are in site surveying setting out, check and compliance surveys. More demanding are the precise GPS applications in building of special control networks for engineering projects (tunnels, bridges), in deformation measurements, or in long term monitoring of structures.

Satellite technologies have rather large error budget, which makes the reliable accuracy planing more complicated. The accuracy of GPS results depends upon many heterogeneous influencing factors, both random and systematic. Important here are the questions of optimal observation parameters and processing procedures using various standard and advanced software systems. Reliability of results often depends upon objective factors like actual GPS satellites constellation, state of the atmosphere, and observation conditions on the measuring site. Also important is the quality of GPS instrumentation. Investigations are focused on testing and optimizing the GPS observation and processing procedures.

2. GPS ERRORS AND ACCURACY

Although GPS is clearly the most accurate worldwide navigation system yet developed, it exhibits more or less significant errors. The accuracy of GPS results depends upon many heterogeneous factors in relation to purpose of the measurements, GPS observation method, instrumentation, conditions in situ, satellite constellation, and processing technology.

In engineering applications mostly phase data are used. The baselines between a reference and rover stations are computed with help of single, double, or triple differences of phase measurements. Many systematic influences are eliminated in this way. Longer observation intervals eliminate most of the periodical errors. Another possibility (like in classical geodetic methods) is to repeat the measurements in different time, and in different observation conditions. Greater the time span of observations, higher the variations, but better the real accuracy with smaller correlations in the output parameters.

Important for a surveyor is to do proper error preanalysis concerning the surveying method employed. For the purpose it is necessary to know about real accuracy limits, so as to choose technically and economically optimal measuring technology. The accuracies of GPS measurements depend mainly upon system and instrumental errors, both random and

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systematic. Most dangerous are the systematic biases which affect the results external accuracy. Particular GPS error sources and ways of elimination are described f.e. in (Teunissen, Kleusberg 1998).

Processing stage is also important - the proper processing procedure (strategy) must account for most of the biases. Problematic are the covariance matrices of processing results which are underscaled (and therefore not realistic) in most cases. An experience is that the relative information in covariance matrices is more reliable than the scale. That often makes problems in weighting of observations in adjustment, especially if different sessions or GPS and classical measurements are combined.

There are three elementary ways to more informative accuracy evaluations: the comparison of session results (preferably sessions of different days), the comparisons of GPS results vs. "ground truth" (well determined station positions), and the results comparison of GPS and other (classical) geodetic measuring methods.

In present time the data and products of the International GPS Service (IGS) are available. There are two main contributions of permanent stations data. First is the provision of good quality data which can be used for computation of more reliable ionosphere model. Second is the possibility to include the permanent station into the network structure, with gains of easy maintaining a consistent reference frame f.e. for long term monitoring of structures. It may also reduce the survey costs. In near future there will be even the possibility to use data from some permanent stations in real time applications. Some experiments concerning the permanent station TUBO situated at Brno University of Technology were carried out. The products of IGS (precise satellite orbits, clocks) are nowadays of high precision and may be contributing to better accuracy in some cases.

3. LOCAL GPS TESTING NETWORK

Important for reliable accuracy evaluations is a well determined set of points - a local GPS testing network. An example of such testing area is the Sněžník network, which was founded ten years ago as a local experimental GPS network in czech-polish cooperation. Nowadays it is one of the most precise local networks in Czech Republic. Since 1992 several campaigns of GPS, EDM, levelling, gravimetric, astronomical and other measurements had been carried out in the network within several research projects. Brno University of Technology had organized observation activities and carried out the measurements and processing of all campaigns in czech part of the network since 1994.

The network is situated along the Czech - Polish frontier in Králický Sněžník mountain region (northern Moravia), in an area with heterogeneous geologic and geotectonic structure. It is composed of 28 monumented geodetic points and 4 adjoining distant points, all founded in solid rock formations and equipped with forced centring heads. The Czech part of the network includes 12 points situated on both sides of the upper Morava river valley. Highest point is SCZE (Králický Sněžník - 1424 m), lowest point is VLAS (Vlaské - 490 m). Longest baseline is 13659 m, maximal height difference is 977 m.

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Fourteen measuring campaigns were realized during the ten years history of the Sněžník network, with GPS as main measuring technology; in some of the campaigns also other terrestrial geodetic methods were employed. GPS data were observed with dual-frequency receivers LEICA (SR299/399), ASHTECH (Z-XII), TRIMBLE (4000SSE/Ssi). Most of the campaigns were concentrated on measurement of the complete network, some campaigns were dedicated to special experiments. The measurements include wide range of GPS baselines with observation intervals varying from half of an hour to several days sessions, with baseline lengths varying between 0,5 - 13,7 km. Essential is the possibility to combine GPS and terrestrial measurements in some parts of the network. Various experiments were focused on GPS accuracy investigations, combination of different GPS receivers, relations of the baselines measured in sessions of different duration as well as comparisons of results obtained with help of various processing software products. Results of some particular evaluations were published i.e. in (Švábenský and Karský 1999), (Švábenský et al. 2001).

In Table 1 the accuracies of adjusted single coordinate componets of 8 points in southern part of the network are given in respect to fixed point VYHL, together with differences between commercial LEICA SKI and scientific Bernese GPS Software results.

Station	Accuracy [mm]			BERNESE - SKI, L1 solution			
	s (B)	s (L)	s (H)	dX [m]	dY [m]	dZ [m]	
0025 TVDR	1.4	1.4	2.5	-0.0005	-0.0022	-0.0033	
0026 KLEP	0.8	0.7	1.3	0.0010	0.0001	-0.0019	
0027 VYHL	-	-	-	0.0000	0.0000	0.0000	
0028 MALI	1.6	1.9	2.7	0.0037	0.0006	-0.0015	
0029 LOMA	1.5	1.2	2.6	0.0001	0.0003	0.0018	
0030 ROUD	1.4	1.4	2.2	0.0018	-0.0003	0.0000	
0031 VESE	1.0	1.0	1.5	-0.0008	-0.0002	-0.0028	
0120 VLAS	1.3	1.5	2.1	-0.0007	-0.0012	-0.0001	
0201 DMOR	1.2	1.2	1.9	-0.0003	0.0004	0.0004	
Std. dev. (components)	1.3	1.3	2.1	0.0015	0.0009	0.0017	
Std. deviation (mean)		1.6		0.0014			

Table 1: Sněžník Network - Position Accuracy and Differences BERNESE - SKI

Differences between processing results obtained with help of the standard software SKI (SKI-Pro) and the scientific Bernese GPS Software are on a milimeter level and therefore are not significant for networks with baselines under 10 - 15 km. Best results are achieved when the L1 carrier is used in such cases. The results using L3 iono-free linear combination are not satisfactory for such networks and show greater variations especially in cases of shorter sessions. This is due to the higher noise on L3. Mandatory for L1 processing is the use of appropriate ionosphere correction model. There is always the possibility to compute a local

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iono-model using the actual measured data, but often more reliable are global or regional ionosphere model which are available through IGS. The authors have tested for the differences in application of local and global (CODE) iono-model when processing the campaign 2001. There was slight improvement in baseline processing results up to 4 %. Probably the best course is a local model with implementation of the data of neighbouring permanent stations.

The above results were achieved in optimal conditions using one type receivers, forced centring of antennas, and precise measurement of antenna height offsets. When processing sessions with different receivers the accuracies may be lower due to the residual uncertainties in antenna phase centre offsets. Best solution here are the special calibrating sessions in field.



Fig. 1 Slope distance and height difference variations

Fig. 1 shows the variations of the processing solutions on L1 carrier in relation to observation time. Baselines of various sessions in ten years span, of various lengths and height differences were processed with Bernese software. Clear is the dampening of variations for observation times 6 hours and more.

4. REDUCED OBSERVATION SCHEME

Observation times (sessions) must be prolonged to achieve more precise results, what is true for most of the GPS applications. Highest precision needs very long observation sessions (24 hours or more) which are very costly. Therefore it seems useful to investigate the possibilities of shortening the observation time with minimal loss of accuracy. The authors proposed a special measuring procedure which is based on results of experiments focused on the combination of the data observed in shorter sessions, in different day or night periods (Weigel, Švábenský 1999). It shows that it is convenient to combine two (dyada) or better three (triplet) shorter sessions of 1-2 hours duration, measured eight hours after each other. In Table 2 some characteristics are given for height component of baseline VYHL - VESE in experimental geodynamic network Sněžník. The difference between minimal and maximal values of one hour solutions is up to 4 cm in course of 24 hours. In combinations of one hour solutions with eight hours interval the differences are four times smaller.

Accuracy of a baseline	1 hour sessions repeated					
height component	after 1 hour	After 2 hours	after 8 hours	after 12 hours		
s.d. of dyada averages [m]	0,0115	0.0097	0.0063	0.0082		
s.d of triplet averages [m]	0.0097	0.0085	0.0038	0.0074		
s.d. of adjusted triplets [m]	0.0100	0.0087	0.0045	0.0070		

Table 2 : Height difference dyads and triplets (baseline 6155 m , height diff. 273 m)



Fig. 2 Variations of triplets and triplet averages

An example of differences between variations of triplets and variations of triplet averages in relation to triplet intervals for a latitude component is given in Fig. 2. Optimal interval of 8 hours is illustrated here.

5. PRACTICAL EXAMPLES OF ACCURACY EVALUATION

5.1 Improved Vertical GPS Accuracy

For the purpose of quasigeoid investigations a levelling section in direction of its maximal gradient in southeast part of Czech Republic was measured. The levelling lines of I. order going through this region were divided into four individually measured segments, with subsequent lengths 28, 10, 31, and 33 km. The segments mentioned were measured by GPS method in years 1999 – 2001 (campaign PROFILE).

For actual GPS measurements the levelling benchmarks with good observation conditions were chosen, eventually excentric stations were established in near vicinity of the benchmarks. Preferred were benchmarks with bedrock founded monumentations. Distances between single stations were 2 - 3 km. The heights of excentric stations were determined by precise levelling from nearest benchmark. GPS measuring procedure was chosen so that every station was measured 3 times in 24 hours. Duration of one observation session was about 1,5 hour. Station in the middle of section observed continually all 24 hours, stations in one and three quarters of section observed 12 hours (3 times 4 hours). The connecting station

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TUBO observed also 24 hours continually. Time schedule of the observation procedure for one of the segments is given in the following scheme (Fig.3).

The observation scheme permits to combine independent or dependent baselines, with lengths of 2 - 30 km. 15 stations were measured in 24 hours. Measurement of the complete section (102 km, 50 stations) lasted 96 hours, with 6 receivers. Triplet session length 90 min allowed the secure ambiguity solving even in bad observation conditions

As for GPS instrumentation, a dual frequency receivers Leica SR 299/399 with internal antennas were mainly used. Station TUBO was observed with receiver Ashtech Z-18. Observation parameters were: elevation mask 10 deg, recording rate 15 sec.

The measured data were processed with help of two software products – scientific software system BERNESE v. 4.0 (4.2), and standard commercial software Leica SKI v.2.3 (SKI-Pro v. 1.1). All possible combinations of baselines were evaluated with aim to analyse the height accuracy obtained by given technology. For final results the precise ephemerides from CODE Bern, individually computed ionosphere models (interval 4 hours), and elevation mask 10 deg were used. Corrections of the standard troposphere model (Saastamoinen) were not introduced from reasons of short baselines, small height differences, and small amounts of data. Basic solution stems from L1, another from L3 processing, always with all ambiguities resolved. Corrections of antenna phase center offsets had been introduced. The corrections were determined in special calibration sessions.

station	tim	ne inter	val (UT	C)	14.9.	1999				15.9.	1999	
number	6:00	8:00	10:00	12:00	14:00	16:00	18:00	20:00	22:00	0:00	2:00	4:00
	8:00	10:00	12:00	14:00	16:00	18:00	20:00	22:00	0:00	2:00	4:00	6:00
TUBO												
1												
2												
3												
4												
5												
6												
7												
8												
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10												
11												
12												
13												
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Fig. 3 Observation time schedule for individual stations

In Table 3 the differences in results of repeated measurements in the third segment of the levelling section are displayed. Only shortest baseline solutions were used, many other possible baseline combinations were not introduced.

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Base	eline	Baseline	Height		Residuals		Accuracy	
from	to	length	difference	v1	v2	v3	Si	Sh
		[m]	[m]	[mm]	[mm]	[mm]	[mm]	[mm]
21	22	2300	-53,2	-0,2	5,5	-5,4	5,5	3,1
22	23	2798	19,2	-2,0	5,6	-3,7	5,0	2,9
23	24	2047	24,7	3,8	-2,7	-1,2	3,4	2,0
24	25	3609	-39,1	-0,1	0,8	-0,6	0,7	0,4
25	26	1713	43,8	4,8	-1,6	-3,3	4,3	2,5
26	27	1412	-75,5	2,1	2,3	-4,3	3,8	2,2
27	28	3252	12,2	1,7	-1,4	-0,4	1,6	0,9
28	29	2281	0,1	1,3	1,1	-2,4	2,1	1,2
29	30	2514	-3,2	-6,7	9,3	-2,7	8,3	4,8
30	31	2061	2,1	-2,5	-3,0	5,6	4,8	2,8
31	32	2334	9,5	7,9	-0,2	3,3	10,0	5,8
32	33	1732	1,9	-9,8	5,8	3,9	8,5	4,9
33	34	1354	7,1	-13,4	3,6	9,8	12,0	6,9
34	35	2117	10,6	10,8	-1,1	-9,8	10,3	6,0

Table 3 : Triplet accuracies (Campaign PROFILE)

The residuals were computed in relation to the average of each single triplet. In accuracy column the standard deviations of a single session and the standard deviation of triplet averages are given. Mean standard deviation of a triplet height difference (computed from all segments) is lower than 5 mm.

Partial results were published in (Kostelecký et al. 2001). It is true that the observing scheme is demanding as to precise timing and organisation, but the final results indicate substantial productivity increase in GPS height determination.

5.2 Deformation Surveys of Railway Bridge

Here are presented some practical results and experience in accuracy evaluation in the case of deformation measurements of the railway track (railway line Brno – Hrušovany), at western side of the Ivančice bridge. This railway section has been unstable and problematic in last years, with frequent maintenance needs. Deformation surveys are going on here since 1999. Static GPS method is combined here with classical terrestrial surveys (precise levelling), what enabled some comparisons and evaluation of real GPS accuracy in this case. Measured data were processed with commercial software LEICA SKI-Pro and also with scientific Bernese GPS software.

Ten epochs of deformation surveys since 1999 are completed now. Three LEICA 299/399 dual frequency receivers were used. Static intervals were of 6 - 10 hours duration. Observation parameters were: recording rate 15 sec, elevation mask 10 deg. GPS height differences in each epoch were compared with results of precise levelling.

Fig. 4 shows the time evolution in height of the marker which is situated on west abutment pier of the old bridge. Here the results of GPS and PL are compared. Three processing

variants were evaluated: L1, L3, and L1&L2. Deformations were measured in respect to reference marker in 400 m distance, situated on the other side of the bridge. Differences between commercial and scientific software results were negligible in this case.



It is clear from the Fig. 4 that in cases of near reference station the best results (having smallest differences from precise levelling results which are used as a reference) gives the processing of L1 carrier frequency. Practically on the same accuracy level is the processing of both L1&L2 frequencies. On the other hand, L3 combination here gives distinctly lower real accuracy. It is caused by practical elimination of the ionosphere effect for neighbouring stations, and by higher noise level on L3. For horizontal components the differences are less distinctive, nevertheless again significant.

5.3 Measurement of Instantaneous Deformations of Railway Track

Instantaneous deformations of railway track were measured experimentally by Stop&Go method. Object of measurements was the section of railway line in vicinity of the Ivančice bridge. Five measuring epochs were completed since end of 1999. In each epoch the survey was repeated several times with greater time spacing. Survey parameters were: recording rate 5 sec, elevation mask 10 deg. Reference station was established in 100 m distance. Rover receiver measured the sequence of points which were marked on the rail heads (identical points were measured by precise levelling). Precise survey of the rail track was enabled by use of a special light antenna carrier, which was designed and manufactured at the Brno University of Technology (Švábenský 2001). The purpose of this surveys was to determine the instantaneous spatial changes of railway track geometric position induced by traffic, or caused by maintenance works.

Measuring data gathered in course of nearly two years were used for many various analyses and experiments. One of the first aims was the evaluation of the real accuracy of Stop&Go method. Table 4 shows the accuracy parameters (standard deviations) computed from variations of repeated measurements in each of the single measuring epochs. The values from processing of L1 and both L1&L2 frequencies are indicated (without calibration corrections)

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Epoch	σ _{Lat} [mm] L1 L1&L2	σ _{Lon} [mm] L1 L1&L2	σ _H [mm] L1 L1&L2
1999,96	4,0 3,8	3,5 2,6	7,6 6,2
2000,22	8,0 7,9	3,6 3,7	6,5 6,5
2000,32	3,1 4,3	3,2 2,6	3,3 4,9
2000,89	2,5 4,8	4,0 3,0	8,3 7,0
2001,31	5,4 3,6	3,2 2,5	10,3 8,9
Mean	4,6 4,9	3,5 2,9	7,2 6,7

Table 4: Real Accuracy of Stop & Go Survey

It is clear from the table that there are significant differences in horizontal and vertical accuracy. Another aspect is that the L1&L2 processing gives somewhat better results. Discernible is difference between latitude accuracy (cca 5 mm) and longitude accuracy (cca 3 mm), but it must be reminded that the values are slightly influenced also by horizontal centring errors of the antenna carrier. Height accuracy is about two times lower.

6 CONCLUSIONS

Accuracy management is very important for reliable and economic engineering surveys, especially in cases of GPS engineering applications. Nowadays the GPS technology is widely used for building of local special geodetic networks for engineering projects. Characteristic for such networks are much smaller dimensions in comparison with large regional and global networks. Important for reliable accuracy evaluations is the well determined testing model network. Example of such testing area is the Sněžník network. Large amounts of experimental data had been acquired there in past ten years. The data are exploited for various investigations and analyses with aim to improve/optimize observing and processing procedures.

Limiting for GPS heights exploitation is the exact knowledge of the local geoid, especially in mountain areas. Reduced scheme of observation in triplets was used in the campaign PROFILE focused on determination of the quasigeoid section from the differences of GPS and levelling heights, and the geophysical data. GPS height differences were measured effectively and with higher relative accuracy here.

Deformation measurements usually must be of high accuracy, and are demanding of time and costs. In previous time the classical terrestrial methods were used which in many cases can be now substituted by GPS methods, partially or even completely. Accuracy and reliability of the GPS results depends upon many influencing factors, with atmosphere as limiting. Also important are the observing and processing strategies.

Static GPS applications in long term monitoring of structures can give relative position accuracies of a few milimeters depending upon the observation time and distance of the reference station. The permanent GPS stations may help to maintain the consistent reference coordinate system. Stop & Go method can give accuracies better than 5 mm in horizontal

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components, and better than 10 mm in vertical component if the reference station is in near vicinity.

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BIOGRAPHICAL NOTES

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